

Expected Equity Risk Premium, Stock Market Performance, and Fundamental Uncertainty

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Abstract

This paper explores the combined explanatory power of the ex-ante implied internal rate of return (IRR) and Shiller's CAPE ratio on the ex-post performance of the U.S. stock market over the past century. The findings challenge the informational efficiency assumptions, as well as those of stationarity and ergodicity of stock market performance, by revealing a structural dependence on the historical path of expected returns. The results support the existence of a valuation convention, suggesting that the equity risk premium reflects Knightian uncertainty more than measurable risk, and may explain its upward trend since the 1980s. Provided that this convention persists, future stock market performance may prove to be largely predictable — an outcome that runs counter to the semi-strong-form efficient market hypothesis.

JEL : B1, B5, D53, D81, D84, E44, E47, G01, G02, G11, G12, G14, G35, G41, N20, N22, Q43, Q56.

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Introduction

Understanding the dynamics of stock market performance remains a central and often contested issue in the academic literature—and increasingly, a topic with broader policy implications, although this paper does not address the political dimension. Theoretically, four main approaches offer distinct and sometimes conflicting interpretations.

- The efficient market hypothesis (EMH) maintains that all available information is instantaneously reflected in asset prices, rendering it impossible to consistently achieve excess returns. Although Louis Bachelier first articulated this idea in 1900, it was Eugene Fama who developed its most comprehensive formulation, distinguishing between weak, semi-strong, and strong forms of efficiency.
- In contrast, Robert Shiller and other academics have emphasized behavioral biases and mimetic effects that can lead to the formation of speculative bubbles and subsequent corrections. Shiller's work on the cyclically adjusted price-to-earnings ratio (CAPE) suggests that stock returns exhibit mean-reverting tendencies over long horizons, implying partial predictability and challenging the semi-strong-form EMH.
- The rational expectations framework, formalized by Muth and Lucas, holds that agents incorporate all available information and form expectations that are correct on average. As a result, for some proponents—especially within consumption-based asset pricing models like the CCAPM—the equity risk premium becomes theoretically unnecessary.
- The economics of conventions, most notably developed by André Orléan, contends that markets are guided by collectively shared representations that mitigate fundamental uncertainty. In this framework, asset prices are not anchored to intrinsic values but evolve through shared narratives and expectations. This view resonates with Hyman Minsky's theory of endogenous financial instability, which highlights the self-reinforcing nature of booms and busts.

This paper aims to investigate the potential for a critical synthesis of these perspectives by focusing on the ex-ante internal rate of return (IRR) as a proxy for expected returns. Drawing on a set of historical estimations covering nearly a century, we explore whether the ex-post performance of the U.S. stock market can be explained by the historical trajectory of the ex-ante IRR. The evidence suggests that market returns may be, at least partially, predictable over long horizons—particularly at 9 or 10 years, and possibly even at shorter intervals.

By doing so, this study introduces new evidence that appears to challenge three foundational pillars of classical financial theory: stationarity, ergodicity, and semi-strong-form informational efficiency. The results suggest that stock market performance may follow a dynamic process in which evolving valuation conventions among market participants generate partial predictability of ex-post returns. In this framework, the equity risk premium is interpreted less as a compensation for measurable risk and more as a reflection of *fundamental uncertainty*, in the sense proposed by Frank Knight (1921). We keep in mind the reservations of Fama (1998) and Schwert (2003) regarding the stability of anomalies, hence the emphasis here on convention dynamics rather than invariant regularities, within an adaptive perspective in the sense of Lo (2004), where rationality is contextual and conventions are evolving.

The article is organized into five parts. The first reviews the theoretical foundations and the main contributions of the existing literature. The second describes the empirical methodology and the data used to analyze market dynamics over a long horizon, in the continuity of Shiller's work. The third presents the results obtained and their interpretation. The fourth applies the approach to a shorter one-year horizon, following the idea that shared heuristics structure a valuation convention in which the CAPE and the IRR play complementary roles: the latter structuring ex-ante valuation, the former acting as a signal and psychological threshold. Finally, the fifth part discusses the theoretical implications of these results and proposes a reinterpretation of the cost of capital in the form of an implicit convention, which may be referred to as ICCC (Implied Conventional Cost of Capital).

I. Theoretical Framework and Literature Review

The academic literature on the ex-ante required rate of return on equities, and the equity risk premium it embeds, presents a broad and contrasting theoretical landscape. On one end of the spectrum, the intrinsic valuation approach, initiated by Williams (1937–1938), assumes the existence of a fundamental value for financial assets, anchored in anticipated future cash flows and independent of resale price speculation. This framework, implicitly embraced by proponents of market efficiency—most notably Fama (1970)—postulates that asset prices fully reflect all available information, making any speculation about others' beliefs or future values redundant.

In contrast, Keynes (1936) and Orléan (1999, 2011), emphasizing the notion of fundamental uncertainty, argue that it is impossible to mathematically estimate future returns and values of risky financial assets. Under this interpretation, investors cannot rely primarily on an intrinsic fundamental value and must instead engage in a "beauty contest," perpetually speculating on collective psychology and the speculative value markets may assign to assets in the future. Asset

prices thus result largely from conventions that evolve and may disintegrate. When such conventions are driven by emotion and mimetic behavior, they generate endogenous instability—as described by Hyman Minsky (2008). In the absence of anchoring, valuation becomes a matter of representation rather than objective calculation. Within this framework, the risk premium compensates less for quantifiable probabilistic risk than for Knightian fundamental uncertainty. In this sense, the equity risk premium can be seen as a manifestation of liquidity preference, required to face such uncertainty.

Adding to this divergence is the behavioral critique, particularly from Benartzi and Thaler (1995) and Shiller (1981, 2000). Without denying uncertainty, these authors see the risk premium as primarily a reflection of psychological biases. Furthermore, Shiller provides empirical evidence that long-term stock returns exhibit mean-reversion, incompatible with semi-strong-form efficiency. Using the CAPE ratio developed with Campbell (1988), he shows (1996–1998) that average stock market performance is partially predictable over a ten-year horizon. These findings align with other research outside the behavioral critique, including the predictive power of past returns (Fama and French 1988; Cochrane 2001), P/E ratios (Basu 1977; Lamont 1998), and the book-to-price ratio, or B/P (Fama and French 1995). Like Minsky, Shiller acknowledges mimetic behavior and the possibility of irrational exuberance leading to sharp corrections or prolonged downturns.

The rational expectations approach, as formalized by Muth and Lucas, led Mehra and Prescott (1985) to argue that the equity risk premium may be transitory or null in equilibrium under the consumption-based CAPM framework. Building on this, Jagannathan et al. (2000) tested the hypothesis of a vanishing ex-ante equity premium, providing estimates for the U.S. market from the 1930s to the 1990s.

However, empirical estimates by Dimson (2002), based on a century of ex-post returns, suggest a historically high and possibly persistent equity premium. Dimson views stock market returns as stationary and ergodic over the long run, extending insights originally developed by Edgar Lawrence Smith (1924). This perspective remains common among valuation professionals, who often estimate the cost of equity capital using long-run historical averages from Dimson or Ibbotson (2001), as documented by Bancel (2014).

Yet the same survey also points to the growing use of ex-ante premium estimates based on anticipated IRRs—a method promoted by authors such as Damodaran (2022) and Fernandez (2020), whose widely circulated work on SSRN is heavily referenced by practitioners. If one assumes that a required IRR determines asset value, there is no guarantee that the resulting

premium is stationary or ergodic, as Dimson implies. Expanding the analysis to the early 19th century, as Siegel (1992–2014) has done, suggests instead that ex-post stock returns in the U.S. follow multi-decade cycles—on the order of thirty years—which may exceed investors' planning horizons. This casts doubt on the stationarity of the equity premium and challenges the relevance of centennial averages for valuation purposes.

If stock market performance is path-dependent on the trajectory of the ex-ante IRR, the high realized returns in the U.S. may have been historically favored by 20th-century circumstances. According to Anarkulova et al. (2022), the probability of capital losses in the U.S. appears unusually low in international comparison, suggesting that the so-called *American Century* may be historically unique and not necessarily repeatable—potentially implying a secular decline in future ex-post returns.

Blanchard (1993) also questions the ergodicity-stationarity assumption, based on his own estimates of the ex-ante IRR. He identifies a long-term downward trend in expected returns from the 1950s to the 1980s, following exceptionally high levels in the late 1930s and 1940s. He further observes that deviations from this trend appear correlated with inflation dynamics. Accordingly, he anticipated a prolonged period of low risk premiums in the absence of inflationary pressures or emerging systemic risks.

The decline in ex-ante IRRs from the 1930s to the 1980s has been confirmed by Jagannathan et al. (2000) and Fama and French (2002). However, subsequent research extending these estimates into the modern era suggests that the ex-ante equity premium has been rising since then (e.g., Ke et al. 2014; Mishra et al. 2018; Fairness Finance model¹). While interpreting this trend is complex and likely multifactorial, several hypotheses will be discussed in Section IV.

This shift from an ergodic ex-post view to a more dynamic conception of required returns suggests a reversal of the traditional research perspective: rather than inferring return requirements from realized outcomes, it may be more appropriate to explain ex-post performance using ex-ante return expectations. It also calls into question the existence of a shared valuation convention—not based on speculative beliefs or ergodic averages, but on an implicit convention around anticipated IRR grounded in intrinsic valuation logic. This paper aims to test that hypothesis by analyzing the ex-ante IRR for the S&P 500, as it might have been estimated by investors throughout the past century, and its explanatory power for ex-post market performance.

¹ Fairness Finance is the only model in this study that originates from private research. It will be described in the following section.

The model also incorporates Shiller's CAPE, resulting in a novel approach, to our knowledge, within the existing literature.

II. Data and Econometric Methodology

The internal rate of return (IRR) of a stock is neither a declarative nor a contractual figure, nor one that can be directly inferred from its price—as is often the case with a listed bond. In the case of a plain-vanilla bond, its conditional IRR (in the absence of default)² can be estimated since the cash flows are predetermined on a fixed schedule. This is not true for equities, which may be considered as perpetual instruments (or at least of uncertain duration) and whose returns, even assuming issuer survival, must be estimated. The stock market IRR is therefore implicit by nature and requires a valuation model based on forecasts which, however justified, remain uncertain and potentially subjective.

There are several ways to implement this estimation. One may rely solely on the cash flows expected to be received by the investor until exhaustion—an “intrinsic” approach. Alternatively, one may introduce a terminal resale value, characterizing a more “speculative” approach. These two approaches may also be combined. In this paper, we prioritize the intrinsic method, as it underlies most commercially-used models³ for estimating the required IRR. Moreover, the speculative approach warrants a research agenda of its own, which lies beyond the scope of this study.

Finally, stock market IRR estimates may be constructed either from individual stock IRRs (a bottom-up approach) or directly from aggregate market-level parameters such as total capitalization and aggregate cash flows (a top-down approach). Both methods are represented

² It should be noted that estimating the expected IRR (in the mathematical sense) of a bond is considerably more complex than calculating its conditional IRR based on the cash flows contractually scheduled under full execution. The former requires modeling the probability of default of the instrument or its issuer, as well as the expected loss in the event of such an occurrence. As a result, if contractual cash flows are not probabilized, the conditional IRR inferred from the bond price will embed a risk premium for potential default losses. The same applies to the IRR of an equity security when it is estimated based on cash flows expected under the assumption of issuer survival. This “gross” IRR likewise incorporates a default risk premium and thus exceeds the mathematical expectation of the stock's actual return. For a discussion of the equivalence between default premium and expected cash flows, as well as the modeling of default risk, see: Clère and Marande (2017).

³ This section considers market models inspired by the work of William Fouse, which allow for the estimation of one or more equity risk premia based on an aggregated ex-ante IRR of listed companies, calculated from their market prices and cash flow forecasts. According to the history of modern finance outlined by Peter Bernstein, the first ex-ante market line was computed by William Fouse in 1970 using stock betas and ex-ante IRRs estimated via a dividend discount model. This approach was first implemented by the discretionary portfolio management division of Wells Fargo Bank (see Peter Bernstein, 1992, ch. 13). In France, similar models have been developed and commercialized, notably by Associés en Finance (from the 1970s), Détroyat Associés (in the 1990s), and later Fairness Finance (in the 2000s). Other models have emerged more recently, such as Valphi and the CCEF model. Most banks also produce their own estimates of the market IRR.

among the estimation sources selected for the historical reconstruction presented in this paper (see below).

These sources thus allow for the reconstruction of a long-term time series of estimated IRRs for the U.S. stock market as a whole. This compilation, in a way, avoids reliance on any proprietary model. However, it introduces a degree of heterogeneity. Accordingly, this paper is conceived as a first step aimed at assessing the relevance of initiating further research to determine the most appropriate model for constructing a consistent series that would best reflect investors' expectations of equity returns since the early twentieth century.

The initial step in this validation process is to examine the overall profile of the reconstructed series. As will be shown, it displays a long-term downward trend, on which the various sources appear to agree—suggesting that the IRR has not been stationary. If, indeed, it reflects the level of return actually required by investors, this would imply that the market's ex-post performance has been positively influenced by it, and that such performance is itself neither stationary nor ergodic, but instead path-dependent.

Demonstrating that the intrinsic IRR is the most faithful proxy for the shared perception of expected equity returns is no easy task. Commercial IRR estimation models designed for use by investors and valuation professionals, such as Fairness Finance, primarily serve to justify a security's value based on a set of parameters. Their goal is to articulate a broadly accepted valuation convention that may evolve over time. This does not, however, guarantee any empirical relationship between these IRRs and market performance—let alone validate the rational expectations hypothesis, which holds that economic agents' forecasts are, on average, accurate.

Testing the accuracy of IRR models for asset pricing both in and out of the sample from which they were derived is a subject in its own right and falls outside the scope of this article. Instead, we adopt a downstream empirical approach, aiming to identify a structural relationship between the estimated ex-ante IRR and ex-post market performance. To the extent that the interaction uncovered aligns with the implications of the initial hypothetico-deductive reasoning⁴, the objective is to determine whether the internal rate of return (IRR) required at the time of investment significantly anticipates the ex-post performance of the S&P 500 index, and whether this relationship follows a dynamic governed by recurrent behavioral patterns over the studied horizon — approximately one hundred years. If this were the case, it would not only suggest

⁴ Specifically, synchronous changes in the IRR and stock market performance are at least negatively correlated—or, equivalently, changes in the IRR and in valuation multiples such as the P/E ratio are similarly negatively correlated.

renewed evidence of investors' use of this tool, but also invite reflection on its practical relevance and potential role within their decision-making processes.

Without presuming the predictability of the future or ruling out the possibility of rational expectations, we will limit ourselves here to positing the existence of a *convention*, the rationality of which — described here as *procedural* — we will attempt to characterize. This convention may serve to compensate for fundamental uncertainty while allowing for the estimation of risk linked to certain quantifiable phenomena.

The aggregate internal rate of return (IRR) required ex-ante for the S&P 500 (formerly the S&P Composite) is primarily estimated here based on academic research, allowing the reconstruction of a long time series spanning from 1930 to 2022. These estimates originate principally from three academic sources, each reflecting a distinct school of thought. A fourth estimate comes from private research. Although the underlying models have become increasingly sophisticated over time, they can all be traced back to the model of Gordon and Shapiro (1956):

$$P_t = \frac{E(CF)}{k_t - g_t} \Rightarrow k_t = \frac{E(CF)}{P_t} + E(g_t) \quad 1)$$

Therefore, the index price P_t at the time of investment depends on three parameters: the expected cash flow $E(CF)$ — more precisely, a normative or smoothed cash flow — its growth rate g_t and the discount rate k_t , which corresponds to the IRR required at the time of investment. The ratio of expected cash flow to price can be interpreted as an effective dividend yield, provided that, on average, dividends do not deviate significantly from free cash flow and that the calculation is not distorted by share buybacks (see below).

Despite secondary differences, these IRR estimates are all based on an intrinsic valuation model that could be described as relatively “anchored” to the average profitability of firms observed at the time of estimation. Growth expectations are more subject to debate, but not to the extent that they create a structural break between the four selected series (which do not overlap across the entire period under review).

- Jagannathan et al. (2000): This first source provides average decadal estimates of ex-ante IRRs from the 1930s through the late 1990s. This constraint initially directs the analysis toward ten-year average stock market performances, which limits the number of independent observations but has the advantage of reducing statistical noise compared to annual data. The authors rely on the Gordon–Shapiro formula, applied to the dividend yield calculated in aggregate terms on the S&P Composite and later on the S&P 500 index, without accounting for share buybacks — a practice that only became widespread

in U.S. markets starting in the 1980s, according to Zeng and Luk (2020). They use as a growth rate the annualized geometric average of U.S. nominal GDP expansion over a 70-year period (6.72%), considered representative of investor expectations. This approach is open to debate, as it reflects an assumption rooted in rational expectations theory — which, taken to its extreme, attributes to agents the ability to foresee macroeconomic parameters. Notably, the authors also test a second growth scenario based on a lower rate (5.19%), corresponding to the historical growth of dividends per share for index companies. This alternative specification leads to negative equity risk premia in the low-return environment of the 1980s. For this reason, we do not retain it here: according to the authors themselves, a negative premium is incompatible with an efficient market — thus, to claim otherwise would amount to squaring the puzzle.

- Ke et al. (2014): This second source provides annual estimates of the ex-ante IRR for the period 1974–2012. The model used is based on the excess return approach, where value creation is measured as the difference between the expected return on equity (ROE) and the cost of equity capital. This differential, viewed as an economic rent, is projected over time to derive the firm’s valuation. The model requires explicit five-year forecasts of book equity and earnings, assuming that ROEs gradually revert to their historical average and that long-term growth converges toward the rate anticipated by the Federal Reserve for the U.S. economy. A terminal value is then computed to perpetuate the excess return beyond the forecast horizon. The authors offer two aggregation methods (aggregate cash flows versus weighted averages), but we adopt the aggregate approach here to ensure consistency with the other data sources and reduce estimation discrepancies. Although this model is robust at the aggregate level, it does not incorporate firm-specific strategic analysis. Instead, it relies on a simplified forecasting structure, driven largely by the assumed convergence toward historical norms — with the growth path derived from the Fed’s projections as the only forward-looking element.
- Mishra et al. (2018): This third source provides annual estimates of the ex-ante IRR over the period 1991–2012, covering 22 years of observations. The authors employ four models drawn from the academic literature, all based on the discounted value of excess returns. Among these, the model by Claus and Thomas (2001) — which is formally identical to that of Ke et al. (2014) — is retained here due to its lower dispersion and its proximity to the median results. Unlike Ke et al., however, Mishra et al. rely on earnings and book equity growth forecasts sourced from the IBES consensus, i.e., forward-looking estimates provided by sell-side analysts. Moreover, the long-term growth assumption is

derived from firm-specific earnings expectations, rather than from GDP growth forecasts published by the Federal Reserve. These differences introduce an upward bias into the estimates, with IRRs systematically higher — by an average of +2.2% — than those produced by Ke et al. While the model retains the same assumption of mean-reversion in normative ROE beyond the five-year horizon, the use of analyst forecasts, although it improves forecast granularity, rests on assumptions whose optimism is well-documented in the literature (see Easton, 2007; Pevzner et al., 2016; Yardeni, 2022). As a result, caution is warranted when interpreting these IRR levels, which likely constitute an upper bound on the returns expected by the market.

- Fairness Finance is the final model incorporated in this study and the only one originating from private research⁵. It aims to estimate the ex-ante IRR for individual listed equities (in both Europe and the United States) based on five-year forecasts of free cash flows to equity, incorporating sector-specific adjustments and explicitly accounting for changes in financial leverage. The model is characterized by a detailed cash flow modeling approach, the inclusion of over 60 industry sectors, and an aggregation of the overall IRR through capitalization-weighted averaging. For the U.S. model, terminal growth is discounted by 25% relative to the five-year nominal GDP growth forecasts provided by the IMF, in order to correct for analyst optimism — a practice reportedly common among valuation professionals, according to the experience of the model's designers. Two main biases are quantified: the optimism bias associated with analyst forecasts (estimated to overstate the aggregate IRR by approximately 2 percentage points), and the non-probabilization of cash flows in case of default, which introduces an additional upward bias of around 1 percentage point. As a result, the gross aggregate IRR is estimated at around 9%, but the adjusted expected return would more likely fall in the 6% to 7% range, depending on whether index-based investing is assumed to eliminate default risk — not through diversification per se, but through the dynamic rebalancing of the portfolio, which systematically excludes defaulting firms by weighting constituents in decreasing order of market capitalization.

These data are presented in Table 1 below. The resulting series consists of nine decadal averages of the estimated ex-ante IRR, computed as the mean of the four available sources (not all of which are available for every decade). These ten-year IRRs span from the 1930s to the 2010s, with a tenth truncated average covering only the first two years of the 2020s. A detailed

⁵ Developed in 2016 as part of a research and development project at the Paris-based firm BM&A, under the supervision of Pierre Béal (Partner, Head of Valuation), the model was implemented by Stéphane Marande and designed by the author of the present article.

comparison of the differences between the methodologies lies beyond the scope of this article. Nevertheless, despite the heterogeneity of approaches, a certain convergence of estimates can be observed, along with a secular trend that will be discussed further in section IV.

Table 1 :

Decade	Jagannathan	Ke	Mishra	K+M+FF*	Average
1930s	11,9%				11,9%
1940s	13,6%				13,6%
1950s	12,0%				12,0%
1960s	10,0%				10,0%
1970s	10,9%	12,1%			11,5%
1980s	11,3%	11,8%			11,6%
1990s	9,4%	8,8%	10,7%		9,6%
2000s		8,4%	9,3%		8,8%
2010s				9,1%	9,1%
2020s				9,1%	9,1%

* Average of Ke, Mishra, and Fairness Finance until 2012, followed by an estimate by Fairness Finance (2 years for the 2020s decade).

These calculations are likely to be only marginally affected by the bias introduced by share buybacks, as the only series based on dividends (Jagannathan et al.) ends just as this practice began to expand significantly in U.S. markets. The other methodologies rely on return on equity (ROE) and free cash flow to equity⁶, and are therefore not, or only minimally, impacted by this phenomenon. While one might suspect that the IRR estimated by Jagannathan et al. underestimates the true rate due to the retention of part of the distributable cash flow, comparison with the calculations of Ke et al. suggests that any such bias is offset elsewhere. By contrast, the last two sources are likely affected by an optimism bias of approximately 1 to 2 percentage points, due to the use of analyst forecasts as model inputs. To validate these assumptions, it would be necessary to construct a new, homogeneous IRR series spanning the entire period — a task that lies beyond the scope of this paper.

While the IRR is the primary variable of interest in this study, it is considered in conjunction with Shiller's Cyclically Adjusted Price-to-Earnings ratio (CAPE) to test their combined predictive power on ex-post stock market performance. The definition of the CAPE is presented in Shiller (1998) and summarized in Appendix 1. Details on its computation are available on the companion website to his book *Irrational Exuberance* (2000).

⁶ Or *distributable cash flow*, i.e., net income adjusted for the impact of growth on invested capital (i.e., the gap between capital expenditures and depreciation, as well as changes in working capital), while also accounting for the portion of this financing constraint covered by financial debt.

As noted by Blanchard (1993), there is a persistent interaction between inflation and stock market performance, which justifies including this parameter alongside the two previously mentioned variables within a multiple regression framework. The inflation data used correspond to the Consumer Price Index (CPI) published by the Federal Reserve Bank of St. Louis.

A fourth explanatory variable is the real per capita GDP growth. This variable is sourced from the Maddison Project database up to 1981, after which it is drawn from the IMF database.

When referring to long-term U.S. Treasury bond yields (T-Bonds), the data correspond to 20-year maturities from 1930 to 1959, with decadal averages taken from Jagannathan et al. (2000). From 1960 to 2021, the data correspond to 10-year maturities, with yields obtained from the Federal Reserve Bank of St. Louis database.

These variables serve as the explanatory inputs in the econometric model used to account for ex-post stock market performance. It should be noted that, except for the CAPE ratio, all variables are expressed in first differences, so as to capture their changes prior to the investment date. The CAPE is the only variable retained in absolute terms, as it reflects the market valuation level at the time of investment.

The dependent variable is U.S. stock market performance, proxied by the total return of the S&P 500 index from 1957 onward and of the S&P Composite before that. Each annual return includes reinvested dividends. This data is taken from the website of Professor Aswath Damodaran.

The regression function used to model ex-post stock market performance based on information available at the time of investment takes the following form:

$$R_{t+h} = \alpha + \beta_1 \cdot \Delta IRR_t + \beta_2 \left(\frac{CAPE_t}{100} \right) + \beta_3 \cdot \Delta Inf_t + \beta_4 \cdot \Delta Rg_t + \varepsilon \quad 2)$$

Where:

- R_{t+h} denotes the annualized logarithmic ex-post return of the S&P 500 index over horizon h , expressed as a percentage;
- ΔIRR_t represents the change in the average implied IRR between the two decades preceding the investment date t ;
- $CAPE_t$ is the cyclically adjusted price-to-earnings ratio (CA-P/E) for the U.S. stock market, as provided by Robert Shiller, measured at date t . It is divided by 100 to align with the scale of the other variables;
- ΔInf_t denotes the change in the average inflation rate observed between the two decades preceding date t ;

- ΔRg_t represents the change in real per capita GDP growth in the United States over the same period;
- α is the intercept of the model, β_1 through β_4 are the regression coefficients, and ε is the error term.

The results of this regression analysis are presented in the following section.

III. Regression Results

As the overall behavior of the IRR series is discussed in Section IV, the following presents only the regression results related to the IRR and the CAPE ratio, which are the primary variables of interest, along with inflation and per capita GDP growth, as additional explanatory variables.

Shiller's CAPE — calculated ex-post based on the ten-year average of inflation-adjusted earnings (see Appendix 1) — has long been recognized as a valuation metric and a leading indicator of ten-year stock market returns. It appears, however, that the prior change in the required IRR before the investment date significantly enhances the model's explanatory power, as will be shown below.

As indicated above, the first step is a validation exercise aimed at analyzing the interaction between the IRR and the CAPE ratio. The underlying hypothesis is that, all else being equal, an increase (decrease) in the required IRR should correspond to a simultaneous decrease (increase) in equity prices adjusted for long-term economic growth — an adjustment that the CAPE ratio is specifically designed to approximate. Once this relationship is confirmed, the analysis then turns to the respective effects of the IRR and the CAPE on ex-post stock market performance.

1. Interactions Between Fundamental Indicators: CAPE and IRR

A first linear regression using ordinary least squares (OLS), with the CAPE ratio regressed on the IRR (based on synchronized data), reveals a negative correlation between the two variables, as expected. Over the period 1930–2020 — yielding ten observations for each variable — a regression of the decadal average CAPE on the ex-ante IRR and the long-term interest rate explains 94.5% of the variance in the observed values (adjusted R^2 of 93%).

The linear fit can be expressed by the following equation:

$$\overline{CAPE}_{t+10} = - \underbrace{371,7}_{p=2 \times 10^{-5}} \cdot \overline{IRR}_{t+10} - \underbrace{62,3}_{p=1,7\%} \cdot \overline{Rate}_{t+10} + \underbrace{61,7}_{p=10^{-6}} \quad 3)$$

Where $\overline{CAPE}_{t+10}$ denotes the average value of Shiller's CAPE ratio over the decade beginning at time " t ", $\overline{IRR}_{t+10}$ et $\overline{Rate}_{t+10}$, refer respectively to the average ex-ante IRR and the average yield on U.S. Treasury long-term bonds over the same ten-year period.

All coefficients would be statistically significant, at least at the 1.7% level, and the overall model would be significant at the 0.01% level, if the residuals conformed strictly to a normal distribution. In fact, the average CAPE for the 1990s is the sole outlier that deviates meaningfully from the model's prediction — a deviation that echoes the suspicions of irrational exuberance raised at the time by Chairman Greenspan and by Robert Shiller. Even so, the empirical results suggest a strong interdependence, consistent with the existence of a shared valuation convention in which both the IRR and the CAPE may hold a central position.

This interaction justifies the joint inclusion of both variables in the subsequent analysis, which aims to model ex-post stock market performance. The focus now turns to assessing their individual and combined explanatory power with respect to future returns observed over different investment horizons.

2. Modeling Ex-Post Stock Market Performance

The second stage of the analysis consists in empirically testing the explanatory power of variables known at the time of investment — in particular, the change in the ex-ante IRR and the level of the CAPE ratio — with respect to the ex-post return of the U.S. stock market index. This performance is measured as the annualized total return (including reinvested dividends) observed over a 9-year horizon starting from the investment date. This horizon is the one that maximizes the model's explanatory power and is therefore one year shorter than the 10-year horizon used by Shiller (1998).

The approach combines a valuation indicator at time t , namely the CAPE ratio, with pre-investment variations — computed as first differences — of the following fundamental variables: i) the internal rate of return (IRR), ii) the inflation rate, and iii) per capita GDP growth.

As previously noted, the model uses changes in decennial averages. All variables are known at the time of investment and are intended to "explain" future returns. The linear adjustment using ordinary least squares (OLS) yields the following regression equation:

$$R_{t+9} = \underbrace{0,2}_{p=2\%} + \underbrace{4,61}_{p=8,6\%} \cdot \Delta IRR_t - \underbrace{0,39}_{p=9,5\%} \left(\frac{CAPE_t}{100} \right) - \underbrace{1,37}_{p=16,5\%} \cdot \Delta Inf_t + \underbrace{1,3}_{p=34\%} \cdot \Delta Rg_t \quad 4)$$

The economic meaning of the variables can be outlined as follows:

- The coefficient associated with the CAPE ratio bears the expected negative sign. It is interpreted as an indicator of market expensiveness, consistent with the interpretation long proposed by Robert Shiller. The December 1999 CAPE value exerts a strong influence on the results, it being noted that this effect is likely amplified by the limited sample size (seven observations);
- First-difference variation in the IRR observed prior to the investment is associated with a positive coefficient. In other words, a prior increase in the required IRR — potentially linked to a decline in the CAPE ratio — tends to support stronger subsequent market performance. Conversely, a decrease in the IRR tends to dampen future returns.;
- Variation in inflation here is negatively correlated with stock market performance. The prior variation in the consumer price inflation rate appears to affect interest rate expectations, and consequently, expectations regarding asset returns and their value, which could decline in the event of interest rate hikes.
- Variation in per capita GDP growth can here be interpreted as a leading indicator of demand and investment, potentially influencing expectations regarding future cash flows. This aligns with the findings observed by Siegel (2014) on the long-term history of stock returns.

Linear regressions conducted on these variables show that combining the CAPE ratio with the prior change in the IRR significantly improves the model's explanatory power compared to using the CAPE alone:

- the CAPE ratio alone explains 74% of the variance (adjusted R^2 of 69%);
- the ex-ante IRR alone explains 71% (adjusted R^2 of 65%);
- combined, the two indicators explain 87% (adjusted R^2 of 79.8%);
- adding the changes in inflation and GDP growth brings the explanatory power to 95.9% (adjusted R^2 of 87.7%).

The Shapiro–Wilk test is consistent with the hypothesis of normally distributed residuals. Furthermore, the Durbin–Watson test does not indicate the presence of residual autocorrelation, despite potential concerns due to overlapping explanatory variables, a consequence of the fact that, although expressed in first differences, the variables still rely on overlapping ten-year windows to compute changes in decadal averages. Overall, the coefficients are jointly significant at the 8% level (F-test). The coefficients associated with the change in IRR and the CAPE ratio are individually significant at the 10% level (t-test). While the other two variables appear to be less significant, their inclusion improves the proportion of variance explained — as adjusted for

the number of explanatory variables — and does not violate the principle of parsimony, as confirmed by a lower Akaike Information Criterion (AIC). That said, the regression shows elevated Variance Inflation Factors (VIFs), though none exceed the critical threshold of 10, suggesting moderate but acceptable collinearity.

The regression residuals reveal two major influential points: the periods 1980–1988 (under Federal Reserve Chairman Paul Volcker) and 2000–2008 (covering the dot-com bubble and the beginning of the subprime crisis). Omitting either of these periods would alter the model’s coefficients, although it would not reduce the adjusted coefficient of determination. This observation suggests that the sample has reached a critical size, making the inclusion or exclusion of data points potentially impactful⁷.

Finally, the outperformance observed between 2010 and 2018 relative to the modelled value raises a key question: are we witnessing the emergence of a new valuation convention, driven by the growing dominance of the MAMAA+Nvidia⁸ in market capitalizations, or is this merely a transitory effect linked to the unconventional monetary policies implemented in the aftermath of the 2008 crisis? This question calls for further analysis, which will be addressed in the following section.

3. On the Possibility of Outperformance During the 2010–2018 Period

This alternative model seeks to test whether the previously observed outperformance is primarily explained by a lasting structural shift — such as a new valuation convention linked to the dominance of the MAMAA+N companies — or whether it rather results from a temporary windfall effect stemming from the unconventional monetary policies implemented after 2008 and continued in the years that followed the crisis.

To this end, the approach consists in introducing into Equation (4) variables that are synchronized with the observed market performance and may have influenced it — as well as

⁷ Reconstructing a homogeneous series of ex-ante IRRs by increasing the size of the training sample could potentially enhance the robustness and stability of the model, as annual IRR data would allow for the multiplication of decennial return series. However, the overlap in the dependent variables could lead to increased residual dependence, which may require additional adjustments — such as multivariate Newey–West procedures or temporal clustering methods — to ensure residual independence.

⁸ MAMAA is the acronym increasingly used to refer to the five largest publicly traded U.S. technology firms by market capitalization—typically exceeding USD 1 trillion (i.e., over 10¹² dollars): Meta (formerly Facebook), Apple, Microsoft, Amazon, and Alphabet (Google's parent company). To this leading group, one should add Nvidia, a key producer of components widely used in generative AI. Hence the use, in this article, of the extended acronym “MAMAA+N.” All six firms belong to the information and communication technology sector (ICT), contributing to a notable degree of sectoral concentration within major stock indices. In Europe, the acronym GAFAM (Google, Apple, Facebook, Amazon, Microsoft) is more commonly used to designate these digital giants.

the performance of the MAMAA+N stocks. Specifically, the change in the inflation rate and the change in the interest rate are included. These variables are thus contemporaneous with the explained market performance, rather than known prior to the investment. This leads to an equation that relates the ex-post market return R_{t+9} to three leading indicators computed at time t ($\Delta IRR_t, \Delta Rg_t, \frac{CAPE_t}{100}$) and two "contemporaneous" variables (δInf_{t+9} et δrf_{t+9}):

$$R_{t+9} = \underbrace{0,22}_{p=0,4\%} - \underbrace{0,6}_{p=27\%} \cdot \Delta IRR_t - \underbrace{0,65}_{p=2\%} \left(\frac{CAPE_t}{100} \right) + \underbrace{0,3}_{p=23\%} \times \Delta Rg_t \\ - \underbrace{1,3}_{p=7,5\%} \cdot \delta Inf_{t+9} - \underbrace{0,7}_{p=7,6\%} \cdot \delta rf_{t+9} \quad 5)$$

Compared to Equation (4), the introduction of the two synchronized variables reduces the contribution of the change in the IRR and reverses its effect, while increasing the weight of the CAPE ratio. Thus, the leading indicator based on the IRR could, in principle, be replaced with forecasts of changes in inflation and long-term interest rates—assuming the development of a suitable forecasting model proves feasible. Moreover, it is observed that the ex-ante change in the IRR (ΔIRR_t) partially explains (with an r^2 of 55%) the ex-post change in inflation (δInf_{t+9}), suggesting that the market's anticipated IRR could serve as a leading indicator of future inflation.

This model exhibits a high level of explanatory power, with an adjusted R^2 of 99.8%. Moreover, it eliminates any indication of stock market outperformance during the 2010–2018 period. Such a result suggests that the 11% annualized return observed during that decade may have been driven more by the Federal Reserve's accommodative policies and exceptionally low inflation, rather than by a lasting shift in the valuation regime applied to the champions of the digital revolution.

IV One-Year Market Predictability versus Shiller's Ten-Year Horizon

The second part of our complementary analyses focuses on the forecasting horizon of stock market performance. Here, we highlight results suggesting that performance may also be partially predictable over a shorter horizon—reduced to one year—by adapting the modeling approach initially developed for the nine-year horizon.

We rely on models designed to replicate market dynamics by explaining ex-post performance with variables that are, as much as possible, available at the time of investment. In a first step,

the model incorporates the required internal rate of return (IRR) as the explanatory variable of interest. Given the close association between IRR and Shiller’s cyclically adjusted price–earnings ratio (CAPE), we then examine the possibility of constructing an investment heuristic independent of IRR, drawing on data that are likely easier to obtain. This approach makes it possible to expand the learning sample and implement tests outside the training dataset.

1. One-Year Ahead Predictability Based on the Ex-Ante IRR

From 1974 onward, the availability of annual data allows for testing the explanatory power of a one-year model, as opposed to the nine-year horizon previously used and the ten-year horizon proposed by Shiller (1998). Initial results suggest that short-term predictive power does indeed exist, particularly when using the IRRs derived from the Fairness Finance model, which help explain the annual returns observed between 2015 and 2022 — eight observations, with no overlap in the dependent variable. This short-horizon finding connects with the behavioral finance literature (Thaler, 1999; Hirshleifer, 2015) and runs counter to the semi-strong efficiency hypothesis, while remaining consistent with an adaptive perspective in the sense of Lo (2004).

The model is based on a regression of stock market performance on the IRR and Shiller’s CAPE, both measured at the time of investment, denoted respectively as IRR_t and $CAPE_t$. Given their temporal precedence, a causal interpretation may be warranted if the estimated coefficients are statistically significant and align with theoretical expectations regarding their sign.

A third pre-investment variable, denoted $\Delta\{PE\}$, is defined as the difference between Shiller’s CAPE at time t and the standard P/E ratio calculated at the same date using the aggregate earnings of index constituents over the preceding twelve months. Both indicators are recorded at the time of investment.

$$\Delta\{PE\} = \frac{P}{E} - \frac{P}{\bar{E}} \mid E = \bar{E} \times (1 + g) \quad \Delta\{PE\} = \frac{P}{E} \times g \Leftrightarrow g = \Delta\{PE\} \times \left(\frac{P}{E}\right)^{-1} \quad 6)$$

Where E denotes spot earnings over the last twelve months, and $\bar{E}$ the average of the last ten annual earnings (inflation-adjusted), including E itself. In most cases, spot earnings exceed the ten-year average, so $\Delta\{PE\}$ reflects the cumulative real earnings growth between the long-term average and the current level — approximately over five years, as a first approximation. It can thus be considered an indicator of the trend in the synthetic earnings of the S&P index. For this reason, it is expected to have a positive effect on stock market performance.

A fourth leading indicator, denoted " $\Delta TL - TC_{t-1}$ ", is also included. It is defined as the one-year change, at the investment date, in the slope of the yield curve (difference between the 10-year T-Bond rate and the 1-year T-Bill rate). Its expected impact is a priori ambiguous, depending on whether one focuses on its interest rate effect or on its signal effect in terms of inflation and growth expectations.

The fifth variable is, by exception, the only one that is synchronized with stock market performance. Denoted gE_{t+1} , it corresponds to the year-over-year growth rate, as of end-June, of the 12-month trailing average of reported earnings for S&P 500 companies, as provided in the dataset compiled by Robert Shiller. It is here assumed to serve as a proxy for the evolution of free cash flow. Its marginal effect is therefore expected to be positive.

It should be noted that the variables are subsequently rendered independent of one another through an orthogonalization procedure, which consists in replacing each explanatory variable with the unexplained residuals from its regression on the other explanatory variables. In this way, only the component of each variable that is independent from the others is retained.

The linear adjustment equation, estimated by ordinary least squares (OLS), is as follows:

$$\begin{aligned}
 R_{t+1} = & \underbrace{1,76}_{p=0,05\%} - \underbrace{25,6}_{p=0,05\%} \cdot IRR_t - \underbrace{5,0}_{p=0,05\%} \left(\frac{CAPE_t}{100} \right) - \underbrace{6,2}_{p=0,2\%} \cdot \Delta TL - TC_{t-1} \\
 & + \underbrace{10,2}_{p=0,2\%} \cdot \Delta\{PE\} + \underbrace{0,3}_{p=0,05\%} \cdot gE_{t+1}
 \end{aligned} \tag{7}$$

The quality of the fit is reflected in an adjusted R^2 of 99.9%, the high statistical significance of all coefficients, and compliance with the usual assumptions regarding the residuals (normality, absence of autocorrelation, and homoscedasticity). Although the year 2020 constitutes an influential point, its exclusion would not substantially alter the model's coefficients or intercept, nor would it reduce the adjusted R^2 of the regression. Given the sample size (eight non-overlapping observations), these results should nevertheless be regarded as indicative, and this analysis as a preliminary step that calls for further work based on a longer series of IRRs.

The variance analysis shows that approximately two-thirds of the information accounting for the one-year return of the market index is already embedded in ex-ante variables known at the time of investment, while the remaining one-third is driven by the ex-post variation in earnings per share.

As indicated above, only earnings growth (gE_{t+1}) and the difference in P/E ratios exhibit positive marginal effects on stock market performance, with coefficient signs consistent with intuitive

expectations. In contrast, the three other variables are associated with negative regression coefficients, suggesting that the market interprets them as risk factors:

- The market multiple ($CAPE_t$) acts here as an indicator of market expensiveness, since it exerts a consistently negative effect on performance regardless of its level. One could nevertheless imagine that sufficiently low multiples might be interpreted as buy signals. It is worth noting, however, that the observation window is limited and the training sample is small (eight observations). Since the multiple is calculated at the time of investment, any significant predictive power would contradict the semi-strong-form efficiency hypothesis, which underpins the theory of rational expectations. Such a result would be all the more noteworthy given that the investment horizon here is only one year — a timeframe much more exposed to statistical noise than the ten-year averages used by Robert Shiller.
- The IRR prevailing at the time of investment appears here to function less as a promise of future return than as a risk variable — an unexpected result suggesting that high IRRs may have tended to adjust downward through subsequent revisions in either the normative level of cash flow or the growth rate.
- Finally, the change in the slope of the yield curve (captured by the pre-investment variable " $\Delta TL - TC_{t-1}$ ") also appears to be perceived more as a rate-related risk factor than as a favorable signal regarding future growth prospects. The mechanism likely at play here is the lagged effect of interest rate increases on investment.

These results are encouraging but must be confirmed through further research — in particular, by being replicated at different points in time and thereby expanding the training sample, which currently consists of only eight observations. It would also be of interest to replace the ex-post earnings growth rate with a forecast of earnings growth. Such a 12-month earnings forecast model is entirely conceivable, given that this type of projection is routinely produced by investment banks with dedicated research departments. While the development of such a model goes beyond the scope of the present study, these initial findings once again suggest that the performance of the world's leading equity market does not conform to the random walk hypothesis or the semi-strong-form efficiency hypothesis.

If this first approach, based on the ex-ante IRR, suggests one-year predictability of stock market performance, it is already possible to extend the analysis by relying on proxies for the IRR, thereby lengthening the historical depth of the series and enabling out-of-sample testing.

2. One-Year Ahead Predictability with Proxies for the Ex-Ante IRR

The availability of a responsive IRR that reflects market heuristics is not necessarily universal, as not all investors have access to the same resources as the portfolio managers of large investment funds. One may therefore hypothesize that substitutes for the ex-ante IRR exist and that these substitutes may prove effective—or at least that there are indications of their use that can be brought to light through an analysis of one-year market dynamics.

Among valuation variables, the first that comes to mind, given its long history and availability, is the price–earnings multiple (P/E) and its possible variants, most notably Shiller’s cyclically adjusted P/E (CAPE). A second candidate is the price-to-book multiple (P/B), which is even simpler to calculate and reflects the productivity of capital. Moreover, when discounted cash flow (DCF) valuation is reduced to a formula inspired by the Gordon–Shapiro model, one observes a close connection between the conditional IRR under survival, the CAPE, and the P/B ratio (see Appendix 3). This makes these multiples natural candidates to serve as substitutes for the IRR as ex-ante explanatory variables of ex-post stock market performance.

To test this hypothesis, we construct a multiple regression model in which the endogenous variable is the one-year ex-post logarithmic performance of the U.S. stock market index (S&P 500), denoted as $g\{P1\}$. The explanatory variables are all available ex-ante and computed at the time of the investment that precedes the ex-post stock market performance $g\{P1\}$. Since they involve the price-to-book multiple (P/B), and given that this synthetic measure has been calculated by S&P only since 1999, it becomes possible to use the quarterly data thus provided to conduct the test.

The period that can potentially be analyzed therefore covers three crises of different natures: the Internet bubble, the systemic financial crisis of 2008–2009, and the COVID-19 pandemic crisis. Of these three, the first was more sectoral in nature, while the latter two proved more universal. Because only aggregate data are available here, the analysis window is restricted to twenty years beginning in December 2003. The analysis thus starts after the bursting of the Internet bubble, which allows us, as will be shown, to address the effect of the financial crisis through an objective dummy variable and its impact on other parameters through interaction terms. Following a similar approach to account for the Internet bubble would require the use of appropriate sectoral indicators, which would need to be identified⁹.

⁹ To remain within the logic of multiples, a sectoral P/E—or more likely a sales multiple (EV/S), which better reflects the valuation of loss-making firms—would probably make it possible to construct an indicator linked to the Internet bubble, by comparison with the multiple of the market index (S&P 500) or with that of the index firms outside the technology sector.

The explanatory variables retained amount to ten and can be grouped into three categories:

a) Fundamental variables, derived from the P/E or reflecting the financial crisis

- The CAPE of the S&P 500 index, calculated at the investment date t , taken from the companion website to Robert Shiller's book *Irrational Exuberance* (2000). In this case, its logarithm is retained, giving the variable denoted $\ln(CAPE_t)$.
- The P/E of the index, also calculated at the investment date, based on the trailing twelve-month earnings. Over the past three decades, except during recessions, the P/E has generally been lower than the CAPE, since it reflects—on average—the effect of capacity growth in the differential between the latest rolling earnings and the ten-year retrospective average of earnings. It is also a more cyclical indicator than the CAPE due to its denominator. To make it more stable, however, the denominator is set equal to the average monthly price rather than the end-of-month closing price. Denoted $PE\mu_t$, the multiple is retained here after a double logarithmic transformation and centering¹⁰. The explanatory variable is thus denoted $\ln(\ln(PE\mu_t))c$.
- The logarithm of the CAPE is comparable to the price-to-book multiple (P/B). Moving to the logarithm brings it to the same order of magnitude, allowing the price-to-book ratio to be compared to a proxy that reflects average profitability. The gap between the two measures can thus provide an indication of the state of market valuation, depending on whether it more closely reflects the stock of capital (P/B) or earning capacity ($\ln(CAPE_t)$). Moreover, the two multiples can be seen as components of the IRR (see Appendix 3), and their difference as a substitute. The variable equal to the difference between P/B and the logarithm of the CAPE is orthogonalized with respect to the two previous variables in order to reduce collinearity. It is denoted $\perp PB - \ln CAPE$.
- The dummy variable for the systemic financial crisis is denoted X is. It is justified by the fact that certain variables see their effects diminish (or even reverse) during the crisis. This probably reflects investor heuristics under uncertainty, since, as expected, they react differently to the same stimuli depending on whether they perceive a crisis situation or a “normal” market phase. The variable equals 1 during a systemic financial crisis and 0 otherwise. It could, to some extent, be set empirically at its most acute phase. To facilitate the determination of its window of occurrence, a more automated approach is chosen, based on three indicators published by the Federal Reserve Bank of St. Louis (FRED database): (i) the VIX stock market volatility index, calculated by Cboe from one-month

¹⁰ Centering here helps reduce collinearity in its interaction with the crisis variable. At a later stage, all variables are centered and standardized to facilitate the interpretation of the model.

index option prices, (ii) the Baa–Treasury credit spread, defined as the difference between the average yield on Baa-rated corporate bonds (Moody’s) and the yield on constant-maturity 10-year U.S. Treasury bonds, and (iii) the TED spread¹¹, measured as the difference between the 3-month LIBOR and the 3-month T-Bill yield, an indicator of risk and liquidity premia in the interbank market. These stress proxies fall within the literature on macroeconomic and policy uncertainty, echoing studies on uncertainty shocks (Bloom, 2009), factor uncertainty (Jurado, Ludvigson, and Ng, 2015), and the economic policy uncertainty index (Baker, Bloom, and Davis, 2016).

The threshold for activating X_{it} is set to select the 10% of occurrences corresponding to the highest values of these risk measures, while accounting for persistence effects—crisis episodes tend to extend over time—and regime-shift effects, reflecting the switch from a “normal” market dynamic to a systemic stress phase. The details of the principles adopted are presented in Appendix 4.

b) Variables reflecting the memory of past stock market performance and threshold effects

- The variable $g\{P - 1\}$ is a continuous indicator reflecting recent stock market performance. It is calculated as an average between the last quarterly performance $g\{P\}_{T-1}$ and the average of the three most recent past annual performances, as recorded at the end of quarters ($g\{P-12\}_{T-1}$, $g\{P-12\}_{T-2}$ et $g\{P-12\}_{T-3}$). This approach may reflect a heuristic based on 1.5 years of past performance while giving greater weight to the most recent information from the quarter immediately preceding the investment.

The following variables are dummy indicators intended to capture potential psychological threshold effects:

- $gY < 0$ equals 1 if the past one-year stock market performance, observed at the investment date, was negative,
- $gT < 50bp$ equals 1 if the stock market performance in the last quarter was below 50 basis points (2% annualized),
- $gT > 5\%$ equals 1 if the stock market performance in the last quarter was above 5% (20% annualized).

¹¹ Since March 2016, the TED spread series has no longer been published by the Federal Reserve Bank of St. Louis. We extend it using the spread between 90-day AA financial commercial paper and 3-month T-bills, which is representative of top-tier financial issuers.

c) Interaction variables with the financial crisis

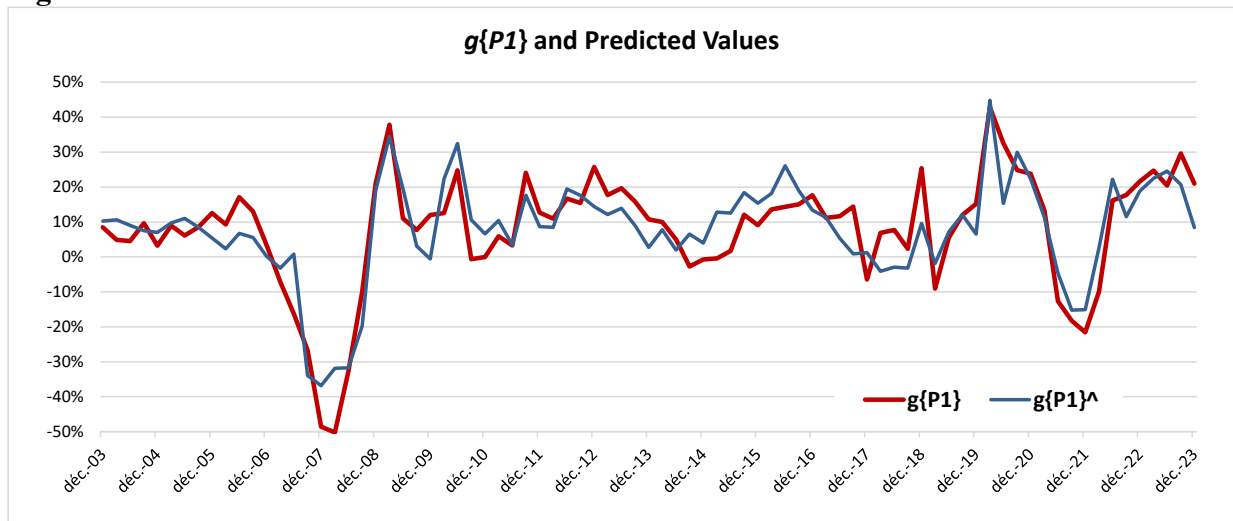
The interaction of variables with the financial crisis dummy (Xis) was tested, and two additional variables were retained:

- The interaction with the P/E ratio, denoted $\ln(\ln(PE_\mu)) \cdot Xis$,
- The interaction with the continuous indicator of recent past stock market performance, $g\{P - 1\}$, denoted $g\{P - 1\} \cdot Xis$.

It should be noted that the selection and construction of these variables—particularly those related to past stock market performance—follow an empirical approach aimed at maximizing the explanatory power of the model. Thus, the core model, consisting of the three valuation variables, the crisis dummy, and a single interaction term (between P/E and Xis), yields an adjusted R^2 of 64%. Adding the continuous past-performance variable $g\{P - 1\}$, which captures autocorrelation in stock market performance, together with the second interaction variable (between past performance and the crisis), increases the adjusted R^2 to 69%. Finally, the inclusion of the three dummy variables designed to reflect psychological threshold effects raises the adjusted R^2 to 73.6%.

To assess more concretely the predictive performance of the model, Figure 1 compares the observed series with the simulated series: the latter reproduces the trends and captures several major turning points.

Figure 1



To facilitate the interpretation of contributions and marginal effects, the variables are centered and standardized. This yields the model whose characteristics and statistics are summarized in the table below:

Table 2

$g\{P1\}$	Observations 81	$R^{2*} = 73,6\%$ $F = 23,3$ $p(F) = 1,6E-18$ $AIC = -392,4$										
		$\ln(\ln(PE_{\mu t})).Xis$	$Xisc$	$LPB-\ln CAPE$	$\ln(\ln(PE_{\mu t}))c$	$\ln(CAPEt)$	$g\{P-1\}$	$g\{P-1\}.Xis$	$gY<0$	$gT<50pb$	$gT>5\%$	Cst
Coefficients		-0,11	-0,18	0,03	0,18	-0,16	-0,12	0,05	-0,04	-0,03	0,03	7,9%
Standard Errors		0,03	0,01	0,01	0,02	0,02	0,02	0,01	0,02	0,01	0,01	0,9%
R ² and Standard Error of Residuals		76,9%	8,3%									
F-Statistic and Degrees of Freedom		23,3	70									
Explained SS and Residual SS		1,62	0,49									
One-Tailed t*, Two-Tailed t		t	t	t	t	t	t	t	t	t	t	t
Student's t-Statistic		4,33	12,85	2,89	7,86	8,01	5,00	4,30	2,74	2,20	2,45	8,56
p(Coef) = 0		4,94E-05	4,4E-20	0,52%	3,3E-11	1,7E-11	4,0E-06	5,5E-05	0,77%	3,13%	1,70%	1,7E-12
Variance Inflation Factor (VIF)		7,9	2,2	1,3	6,3	4,7	6,2	1,7	2,8	2,4	2,3	
Mean Deviation EM{Y}=11,3%		1,0	0,9	0,8	0,4	0,7	0,7	0,6	0,7	0,7	0,4	
Mean Deviation × Coefficient		-11,0%	-16,3%	2,5%	7,7%	-11,3%	-8,6%	3,3%	-3,1%	-2,1%	1,5%	

The sign of the coefficients associated with the fundamental variables (highlighted in green in the table) is broadly consistent with intuition. Thus, the CAPE remains an indicator of expensiveness, even at a one-year investment horizon. By contrast, the momentum effect of the P/E ratio—confirmed by its positive correlation with ex-post stock market performance—is less expected. This effect is strongly attenuated (–60%) during crisis periods, with the interaction variable then taking a negative sign. This interaction can be interpreted as reflecting a rise in the multiple when the financial crisis turns into an economic crisis and is accompanied by a sharp decline in earnings.

The variable corresponding to the difference between P/B and the logarithm of CAPE shows a positive coefficient: when book value exceeds its “profitability-based proxy,” this appears to constitute an empirical buy signal. This result is consistent with the definition of the IRR provided in Appendix 3: a prior increase in the IRR acts as a stimulus for ex-post performance, consistent with the momentum effect already detected in Equation 4.

Moreover, the lagged variable $g\{P - 1\}$ exerts a negative effect on ex-post performance, indicating a tendency toward mean reversion. As the interaction variable shows, this effect is halved during financial crises.

Finally, the dummy variables reflecting psychological thresholds each display a momentum effect on stock market performance.

Diagnostic tests indicate that the residuals follow a normal distribution and show no significant heteroskedasticity. However, the Breusch–Godfrey test reveals significant first-order autocorrelation, which can be explained by the data structure—annual performances calculated over rolling quarterly windows—mechanically inducing overlapping observations. As a result, standard errors were corrected using the Newey–West method, which confirms the significance of the coefficients, including those associated with $gT < 50bp$ and $gY < 0$, which remain significant (at about the 2–3% level). These results are corroborated by an H3 jackknife

correction designed to test the robustness of the coefficients, with even the more marginal ones—particularly those related to threshold effects—remaining significant at the 5% level.

To complement the robustness analysis of the model and to contain the effects of collinearity, a Ridge regression was implemented. Its purpose is to bring the variance inflation factors (VIF) to an acceptable level, which can be achieved here by selecting a penalty value λ , without excessive loss of coefficient significance in the regression.

Thus, a significance threshold of 10% can be preserved up to a penalty value of $\lambda = 2.5$, which brings all VIFs below 3.8 and results in an adjusted R^2 of 61%. With a stricter threshold of 5%, the compromise is reached at $\lambda = 1.3$, with VIFs less than or equal to 5 and an adjusted R^2 of 66%. Finally, minimizing the four-fold cross-validation error ($k = 4$) leads to $\lambda = 0.5$, corresponding to an adjusted R^2 of 70.5%.

In conclusion, Ridge regularization here acts as a complementary robustness test by ensuring a substantial reduction in collinearity. It confirms the soundness of the model and anticipates, through the logic of cross-validation, the more direct assessment of its predictive power that we examine next with the out-of-sample (OOS) forecasting tests.

It should be noted that a complete orthogonalization of the explanatory variables was carried out in order to isolate the independent contributions of each factor and of their most influential combinations. To this end, a modified Gram–Schmidt matrix decomposition was employed, ensuring symmetric orthonormalization independent of variable ordering and thereby eliminating all collinearity.

This procedure highlights the strong contribution to explained variance of the variable $\perp PB - \ln(\text{CAPE})$ ($\approx 50\%$) and of the combination of the P/E ratio and the crisis variable ($\approx 22\%$). It is noteworthy that $\perp PB - \ln(\text{CAPE})$, which is closely linked to the IRR, here proves to have the greatest explanatory power for variance (greater than that of the CAPE). However, since these components are statistical residuals, their economic interpretation remains limited, while the coefficients of other variables cease to be significant.

A more parsimonious specification, reduced to seven such orthogonalized variables, does improve both the AIC and the adjusted R^2 (in-sample), but degrades out-of-sample predictive power. We therefore retain the full model, with ten centered–standardized but non-orthogonal variables, as the reference for robustness assessment and for the implementation of out-of-sample tests.

As recommended by Campbell and Thompson (2008), the robustness of the model is also assessed out-of-sample (OOS). The evaluation was conducted according to two standard

protocols—the cumulative (“expanding window”) approach and the rolling (“rolling window”) approach—while carefully avoiding any look-ahead bias, with each regression using only the information available at the considered date.

In the cumulative approach (starting in December 2003), the model becomes predictive after 56 quarterly observations (≈ 14 years), a threshold beyond which the out-of-sample coefficient of determination, R^2_{oos} , stabilizes between 65% and 75%. This threshold of 56 observations is then applied to the rolling approach, in which the model is recalibrated on a fixed window and used to predict the following quarter’s performance. In this setting, 22 successive forecasts (2018–2023) are produced. The R^2_{oos} then reaches 40.2% over these 22 quarters (≈ 6 years).

For comparison, naïve benchmark models based on the mean of past returns exhibit significantly¹² lower performance: 29.8% for the cumulative historical mean and 35.0% for the five-year rolling mean. These results confirm the robustness of the model and its out-of-sample predictive power, opening the way for potential applications in investment strategies and portfolio management¹³.

This modeling (essentially based on the CAPE) and the preceding one (relying on the IRR) appear to confirm the picture previously outlined at a 9–10 year horizon: the ex-post performance of the U.S. stock market index seems largely predictable from valuation variables available prior to investment, foremost among them the CAPE and the ex-ante IRR. They thus pave the way for future work on the predictability of stock market performance at horizons much shorter than the one envisaged by Shiller (1998).

V Conventional Dynamics, Fundamental Uncertainty, and the Emergence of Threats?

The results obtained suggest that stock market performance can be largely explained, over various horizons (from one year to nearly ten years), by a dynamic based on the interaction between IRR, CAPE ratio, and ex-ante variations in macroeconomic variables such as inflation, long-term interest rates, or economic growth. Other results, seemingly with greater explanatory power due to the inclusion of variables partially synchronized with the endogenous variable, could have been presented. However, given space constraints, it seemed preferable to focus here on models that unambiguously highlight the causal link between the variables and stock market

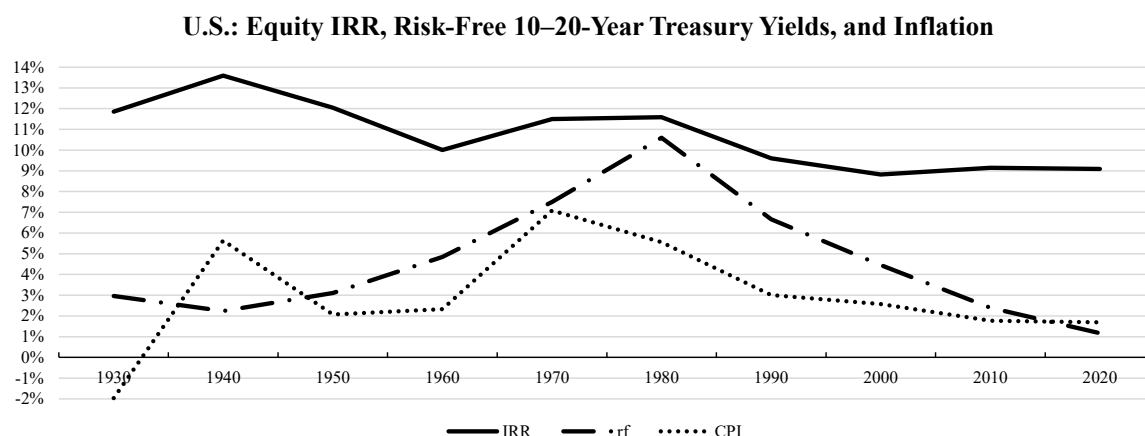
¹² Out-of-sample performance was compared with that of the benchmark models using the Clark–West test. The predictive gain is significant both for the rolling approach relative to the five-year rolling mean ($p = 0.45\%$) and for the cumulative approach relative to the historical mean ($p = 0.07\%$).

¹³ By way of illustration, an equity exposure “tilt” rule could exploit the one-year forecast, with an allocation proportional to the signal (using a clipped-Kelly function) and bounded according to a predefined risk profile.

performance, prioritizing explanatory parameters that can be estimated prior to investment. These results suggest the existence of a shared valuation convention (a behavioral rule), whose persistence is remarkable over nearly seven decades. Indeed, it appears to persist through distinct market phases, which could previously be identified using the CAPE ratio, though without being able to dynamically explain the transition phases (see Appendix 1). Building on the work of Robert Shiller, the explanatory power of these models suggests that stock market movements are not merely the result of chance—contradicting the hypothesis of pure informational efficiency—but instead follow a shared, conventional, and predictable logic. Nevertheless, while this stock market dynamic, based on a combination of factors, transcends discontinuities between market phases, its elucidation naturally leads to questioning its primary cause, which in all likelihood lies in the evolution of risk aversion itself, potentially influenced by partially endogenous factors that now warrant consideration.

Indeed, just as the profitability of the S&P 500 does not appear to follow an ergodic process, its ex-post risk premium would also diverge from such a process, provided the hypothesis is validated that the IRR exerts an influence on market performance. In Figure 2 below, the decadal averages of the estimated required IRR for the S&P 500 are presented and compared to the averages of inflation (CPI) as well as to the yields of U.S. Treasury bonds with maturities of 10 to 20 years¹⁴.

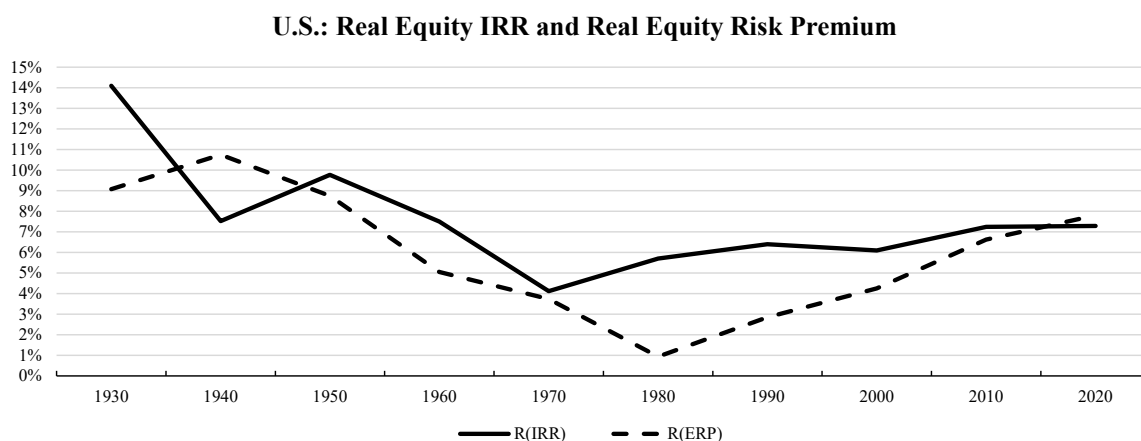
Figure 2



Where k , rf , and CPI respectively denote the decadal averages of the ex-ante IRR of the U.S. market, the risk-free rate, and the change in the consumer price index. The yields on 20-year Treasury bonds prior to 1970, then on 10-year bonds thereafter, as well as the average annual inflation observed ex post at the date of the market IRR calculation, are respectively used for the calculation of the risk premium and real (inflation-adjusted) rates (see Figures 1 and 2).

¹⁴ Until 1969, the yield on 20-year T-Bonds is reported here, followed by the yield on 10-year T-Bonds due to the lack of available data for the entire period. Prior to this date, the yield curve was relatively flat, which minimizes the bias that might be introduced into the analysis by this change in maturity. Ideally, a maturity of 5 to 10 years would be preferred for the entire period in order to reflect the investment horizon generally used in DCF valuation models.

Figure 3



Where $R(k)$ and $R(ERP)$ denote, respectively, the inflation-adjusted IRR of the U.S. market and the inflation-adjusted risk premium.

Figure 3 presents the decadal averages of the IRR and the inflation-adjusted risk premium. The latter is calculated as the difference between the IRR and the yield on T-Bonds, with both figures deflated by the CPI coefficient for the period.

As shown in Figures 2 and 3:

- The long “digestion” of the 1930s crisis appears to have ended in the 1970s, if one considers the IRR net of inflation (see Graph 2). It is estimated to have fallen from 14% in the 1930s to an average of 4% during the 1970s. In the 1980s, the emergence of high real interest rates under Paul Volcker’s tenure at the Federal Reserve may have driven the equity risk premium, also expressed in real terms, to its lowest secular level — around 1% on average.
- Subsequently, the much-discussed disappearance of the equity risk premium, predicted during the 1990s (cf. Arnott & Ryan 2001; Claus & Thomas 2001; Arnott & Bernstein 2002), ultimately did not occur. Indeed, the nominal IRR did not seem to follow the decline in inflation or in interest rates. On the contrary, in real terms, the required IRR has been on an upward trend since the 1970s — a decade marked, let us recall, by the collapse of the Bretton Woods system and two oil shocks. Nor did the “end of history”¹⁵ materialize, as evidenced by the resurgence of geopolitical tensions between East and West, and by the growing threats to the regular functioning of democratic institutions in the West. To the economic and geopolitical crises that have emerged since the beginning of the 21st century must be added the rising awareness of a major systemic risk: climate disruption and the

¹⁵ This refers to the definitive victory of liberal capitalism over communism; see, in particular: Fukuyama, Francis, "The End of History and the Last Man," 1992, Free Press.

impending depletion of cheap fossil energy resources. These threats likely contribute to a general increase in risk aversion and a heightened sense of uncertainty.

- The 300-basis-point decline in the ex-ante IRR that appears to have occurred between the 1950s and the 1990s could largely explain the near doubling (+182%) of Shiller's P/E ratio over that period. Spread over the four decades separating the 1950s and the 1990s, the decline in the ex-ante cost of capital may have generated an unanticipated price increase of approximately 1.5% per year on average, thereby mechanically inflating the ex-post equity risk premium by the same amount. This likely accounts for most of the excess return in ex-post equity performance compared to the expected IRR, which stands at 1.8%¹⁶.
- Lastly, it is worth noting that the nominal expected return on the equity market has remained relatively stable since the 1990s, with decadal averages hovering around 9%. If these calculations are accurate, the stationarity of the ex-ante nominal cost of equity over the past three decades implies that this parameter would therefore not have exerted a trend effect on earnings multiples. Alternatively, should any trend effect have taken place, it would most likely have favored ex-post returns, insofar as the 9% estimate appears to reflect an underlying optimism bias.

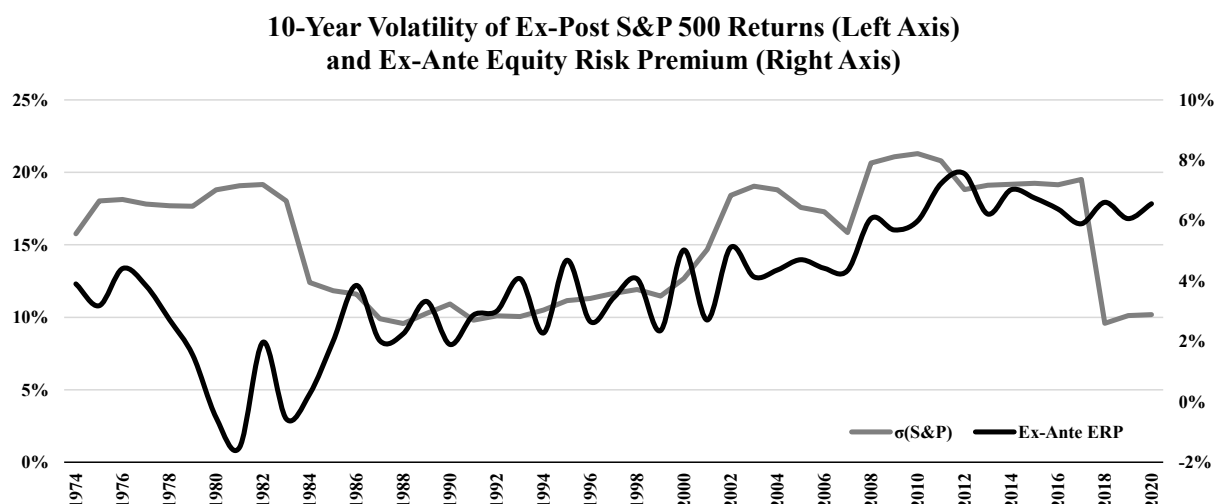
The overall pattern of the historical trajectory of the estimated ex-ante IRR and the resulting risk premium appears to place the concept of fundamental uncertainty, as analyzed by Frank Knight and later taken up by Keynes, at the center of the interpretation of this stock market dynamic.

While the risk premium has seemingly increased in the United States since 1981, there has not been a corresponding rise in risk in the classical sense—that is, in the volatility of market portfolio returns. For the S&P 500, this volatility (calculated on a rolling decadal basis) has remained, since the postwar period, within a range of 10% to 20%.

As suggested by Figure 4, the evolution of the ex-ante equity risk premium does not mirror that of the volatility of ex-post returns on the index. Indeed, when the volatility of the S&P 500 reached its first peak between the two oil shocks, the ex-ante risk premium was likely at its lowest point recorded during the 20th century. At most, one could consider that market volatility may contribute—among other factors—to the level of the equity risk premium, which remains to be demonstrated and is discussed later and in Appendix 2.

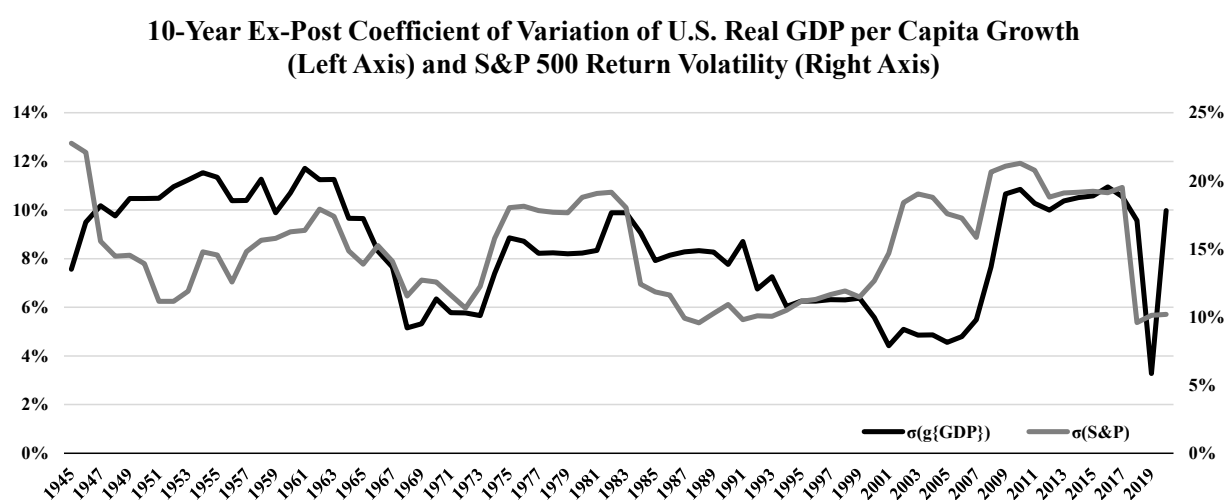
¹⁶ The annualized realized return of the S&P 500 index was 12.7%, compared to an average expected IRR of 10.9%. The effect of the decline in the ex-ante IRR alone — approximately 1.5% per year — is thus not significantly different from the estimated 1.8 percentage point gap between realized and expected return.

Figure 4



The hypothesis put forward by Lettau et al. (2007) is that the equity risk premium reflects the volatility of the underlying economy.

Figure 5



As shown in Figure 5, since the post-war period, it is stock market volatility—not the risk premium—when measured over rolling ten-year periods, that appears to track the scaled¹⁷ coefficient of variation of U.S. per capita GDP growth. It also exhibits specific episodes, such as the dot-com bubble, which therefore seems to be less a macroeconomic phenomenon than a mimetic enthusiasm episode, in Shiller's sense.

Under these conditions, unlike stock market volatility, the ex-ante risk premium can hardly be regarded as a mere reflection of prevailing economic instability. Rather, it should be understood

¹⁷ The coefficient of variation is equal to the standard deviation of ten annual logarithmic changes in per capita GDP divided by the mean of these ten changes. This value is then divided by ten to facilitate comparison with the 10-year volatility of the S&P 500 index.

as the result of perceived future risk. And is it not the very nature of the future to be uncertain, especially when it comes to stock market performance?

The implied equity risk premium, inferred from the ex-ante internal rate of return (IRR), could thus be interpreted not primarily as compensation for a quantifiable probabilistic risk, but rather as a reflection of radical uncertainty about the future. If confirmed, the sharp increase in the ex-ante risk premium since the 1980s would then suggest a growing aversion to a future perceived as increasingly uncertain. This development may be understood through three interpretative lenses:

- The first interpretation is to view the increase in the IRR as an expression of growing concern about the sustainability of the prevailing economic scenario. While the "normative" cash flows incorporated into the models remain based on relatively optimistic extrapolations of past averages, the rise in the risk premium would reflect the fear that this scenario may not materialize.
- Complementary to the previous interpretation, a second reading would suggest that market participants anticipate a long-term slowdown in economic growth — for example, due to resource scarcity or environmental constraints — echoing the scenario outlined in the Meadows Report (1972). In this case, the ex-ante IRR, although it may justify current valuation multiples, would not reflect a market expectation within the usual 7% to 9% range, but rather a significantly lower figure, based on long-term growth assumptions well below those naively extrapolated from past economic trends.
- Finally, a third hypothesis, which is purely technical, is that the risk premium has apparently increased as a result of the unprecedented decline in nominal interest rates and inflation between the 1990s and the COVID-19 pandemic. In this case, the IRR would have reached a floor and would remain “normal,” while nominal interest rates would be abnormally low. Indeed, even in a context where the sense of uncertainty is stationary, the level of stock market volatility would be irreducible, and a floor IRR could then be justified based on a purely stochastic approach to the diffusion of returns. The central tendency parameter would reflect the capital accumulation rate, and the additional expected return would result from stock market volatility. This is a hypothesis considered by Clère (2025), whose main principles are summarized in Appendix 2.

Regardless of the chosen interpretation, it appears plausible that the major long-term shifts in the ex-ante IRR are driven less by observed market volatility than by the underlying principle of uncertainty and the general level of apprehension it induces in response to major geopolitical,

climate-related, technological, or societal imbalances. Moreover, there may exist a lower bound for the IRR that is not accounted for in standard theory and which markets may have only recently come to recognize. If it is indeed a valuation convention that governs stock market dynamics — with the collective perception of fundamental uncertainty as its driving force — this would entail, in a way, an epistemological shift from the concept of *Implied Cost of Capital* (ICC) to that of *Implied Conventional Cost of Capital* (ICCC).

Conclusion

As Eugene Fama (2016) emphasized, despite six decades of research on the cost of capital, it remains difficult to identify a model that faithfully reflects the rational process by which market participants determine stock prices. To contribute to this ongoing inquiry, the performance of the U.S. stock market over a 70-year period was modeled by combining the estimation of the ex-ante implied IRR—as a proxy for the market’s required rate of return—with Shiller’s cyclically adjusted price-to-earnings ratio (CAPE), interpreted as an indicator of perceived market expensiveness. The findings suggest the following interpretations: (i) the U.S. stock market’s ex-post performance does not appear to be stationary but rather influenced by a long-term downward trend in the required rate of return; (ii) however, the ex-ante equity risk premium appears to have declined between the 1930s and the 1970s, only to rise steadily thereafter—a troubling trend that may reflect an increasing perception of uncertainty; (iii) rather than following a random walk, stock market returns seem to be largely explainable by a dynamic process driven by valuation levels and the required IRR; (iv) in this perspective, and insofar as a persistent convention governs asset valuation, market returns may be to a large extent predictable—thereby challenging the efficient market hypothesis and reinforcing previous findings, including those of Robert Shiller, on the predictability of stock returns. In this sense, the IRR and the CAPE are not strictly endogenous to one another: the former structures ex-ante valuation (implied cost of capital), while the latter operates primarily as a signal of expensiveness or as a psychological threshold within the convention.

To the best of our knowledge, this approach is novel: this is the first time that the explanatory power of Shiller’s P/E ratio has been jointly analyzed with that of the ex-ante IRR to explain stock market performance. Each of these two variables may synthesize one of the two opposing perspectives in investing: speculative value versus fundamental value. This approach may thus shed new light on the relatively loose anchoring of stock prices and on the procedural rationality underlying their formation. It may also provide a foundation for interpreting the equity risk

premium as essentially arising from fundamental uncertainty—whose irreducible and shifting nature renders any form of insurance against it ultimately futile.

The analysis of the IRR's predictive power was limited here to the aggregate market level. In doing so, the study aimed to highlight the implicit mechanism of market expectations that underlies investment decisions and valuation. Within this framework, one should not be misled by the apparent predictive value of the 9% nominal IRR that markets have seemingly required on average over the past thirty years. Rather than a promise of future returns, this rate is likely biased upward by overly optimistic earnings forecasts underpinning its computation.

The study of the predictive power of the IRR should, in principle, be able to continue at the aggregate level by reconstructing a homogeneous series of ex-ante IRRs in order to validate these initial results. It could then be extended to the level of individual companies, which should once again allow us to move beyond the traditional analyses based on ex-post returns that have been the norm for the past 60 years. This, in turn, opens up the prospect of a research program centered on the ex-ante risk premium and its application to other risky assets, such as debt securities, startups, or even options considered as risky assets (and not merely as derivatives), thereby dispensing with the need for risk-neutral arbitrage—a practice that some would like to extend even to the valuation of equities via the Merton model.

At the level of equities, such complementary work would notably make it possible to study the forecasting biases that taint the estimation of individual IRRs, and to better understand how investors take these into account in their allocation strategies. It would also allow us to leave behind the rarefied spheres where the visionary genius of a handful of giant companies—architects of our future—unfolds, and instead to apprehend the reality of a more ordinary industrial fabric, which nonetheless constitutes the vast majority of listed companies, even in the United States.

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Appendix 1: Identifying Market Phases Using the CAPE Ratio

The Cyclically Adjusted Price-to-Earnings ratio (CAPE) ¹⁸ was developed by Robert Shiller and John Campbell in the late 1980s, during their research on stock price volatility and the link between the historical average of corporate earnings and the present value of future dividends (1988). This work led to a second publication in July 1996, where the proposed multiple was presented as a leading indicator of stock market overvaluation, particularly in the context of what would later be identified as the dot-com bubble.

The expression "irrational exuberance" was popularized by Federal Reserve Chairman Alan Greenspan in a speech delivered on December 5, 1996, just two days after Robert Shiller had shared this research with him. The analysis was subsequently expanded in a third article (1998), and later integrated into Shiller's book *Irrational Exuberance*, first published in 2000. The book's companion website provides a detailed explanation of the CAPE calculation methodology, which is still updated regularly.

The formulation of CAPE can be summarized as follows:

$$CAPE_t = \frac{P_t}{10} \times \left(\sum_{i=10}^1 E_{t-i} \times \frac{CPI_t}{CPI_{t-i}} \right)^{-1} \quad 8)$$

Where P_t denotes the level of the S&P Composite/500 Index at the calculation date, i.e., the end of month t . The most recent net earnings, E_{t-1} , correspond to the trailing 12-month earnings as reported at the beginning of month t . This annual figure is reconstructed quarterly by S&P from interim financial statements of the constituent companies (which do not all share the same fiscal year-end). For each of the two intervening months, the trailing net earnings of the index are estimated retrospectively by interpolating between the quarterly values. The nine preceding annual earnings figures are similarly reconstructed, going back to E_{t-1} . Each earnings figure is adjusted by the variation coefficient of the Consumer Price Index (CPI). In monetary terms, the earnings over the 10-year period used in the calculation are thereby rendered comparable, which, according to the authors, justifies taking their average¹⁹.

¹⁸ The term CAPE was initially used sparingly by Shiller himself when presenting his work, but it was widely adopted by commentators and the financial press. Acknowledging the term's success and widespread use, Shiller eventually embraced it in subsequent editions of his book *Irrational Exuberance*. Both the academic community and valuation professionals also refer to it as "Shiller's P/E ratio".

¹⁹ In their 1996 article, the authors refer to Benjamin Graham's recommendation for evaluating stock market performance but adopt a 30-year horizon for calculating average earnings—an approach also found in the work of J. Siegel. In their 1998 article and thereafter, both time horizons were aligned and set at 10 years.

In *Irrational Exuberance* (see Introduction, p.7, and Chapter 11), Shiller uses the CAPE ratio to warn against the irrational exuberance of markets, which he identifies as the psychological foundation of speculative bubbles. On this point, his view of investors' herding behavior converges with that of Hyman Minsky (2008). Shiller notes that locally elevated levels of the CAPE ratio have often preceded stock market crashes or periods of stagnation. The inflection points and peaks observed in the time series under study are also identified and commented on by Hyman Minsky in his analysis of the evolution of the U.S. economy and financial markets (during the period common to both analyses).

In other words, the tendency of the multiple to revert to a certain mean appears to result in a negative relationship between stock market performance and the preceding level of the CAPE ratio.

In their 1998 article, Shiller and Campbell report that over the period covered by their calculations—from 1871 to January 1997²⁰, the logarithm of the CAPE ratio accounts for 37% of the variance in the 10-year average annual log return (excluding inflation) of the S&P Composite-500 Index²¹. They observe that at a one-year horizon, the explanatory power of the CAPE is much lower, with an R^2 of only 4%. Mean reversion thus occurs over several years, suggesting a certain capacity of the "system" to return to a trend growth path, through the operation of the Law of Large Numbers, albeit with shifts in the underlying growth regime, as we shall discuss further below.

When these calculations are repeated today using the updated data available on Robert Shiller's website, regressing the 10-year average return yields a slightly higher explanatory power, with a coefficient of determination of 39.7% (adjusted r^2 of 39.6%), as shown in Table 2 below. This slight improvement likely reflects minor differences between the current series and those originally presented to the Federal Reserve Board at the time.

²⁰ To compute the 10-year rolling CAPE and compare it with the index's subsequent 10-year ex-post performance, the common data window for both series is restricted to the period from 1881 to 1987.

²¹ The authors assess the significance of the regression coefficients using the Wald test, employing robust standard errors to account for serial dependence induced by overlapping data (10-year rolling averages). As a result, monthly observations are not independent, rendering the use of conventional tests invalid without appropriate correction. Rather than limiting the analysis to non-overlapping subsamples, the adopted procedure adjusts standard errors to account for this dependence. In Tables 2 and 3, where Student t-tests are presented, the adjustment procedure (Newey-West) has not been applied, although it should rigorously be implemented in the presence of autocorrelation.

Table 2

Annualized Stock Real Return	1881-1987	R ² * = CA/PE	39,6% Cst
Coefficients		-9,4%	31,2%
Standard Errors of Coefficients		0,3%	0,9%
R ² and Standard Error of Residuals		39,7%	4,01%
F-Statistic and Degrees of Freedom		836,5	1271
Explained and Residual Sums of Squares		1,34	2,04
One-tailed t*, Two-tailed t		<i>t</i>	<i>t</i>
Student's t-Statistic		28,9	36,2
p(Coef) = 0		3E-148	5E-210

Table 3

Annualized Stock Real Return	1881-2014	R ² * = CA/PE	28,5% Cst
Coefficients		-7,0%	25,7%
Standard Errors of Coefficients		0,3%	0,8%
R ² and Standard Error of Residuals		28,6%	4,36%
F-Statistic and Degrees of Freedom		640,5	1599
Explained and Residual Sums of Squares		1,22	3,04
One-tailed t*, Two-tailed t		<i>t</i>	<i>t</i>
Student's t-Statistic		25,3	33,8
p(Coef) = 0		4E-119	3E-189

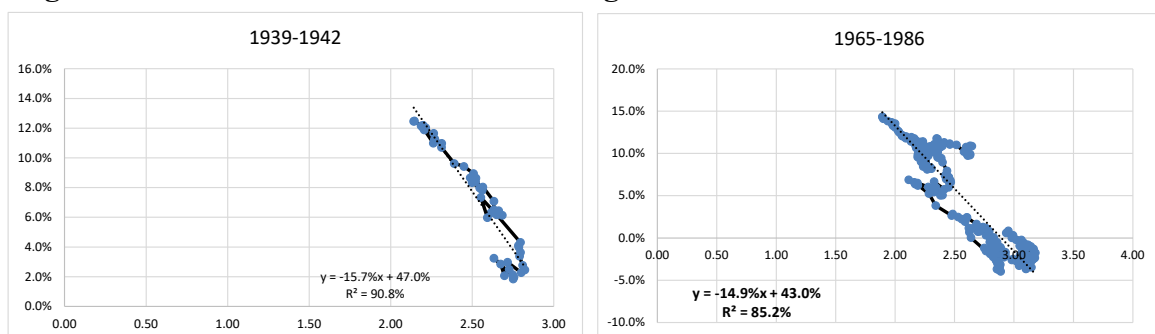
Taking into account 27 additional years up to the end of May 2014, the logarithm of the CAPE explains 28.6% of the variance in the rolling 10-year average of annual log returns on the market. The regression parameters and characteristics are summarized in Table 3 above.

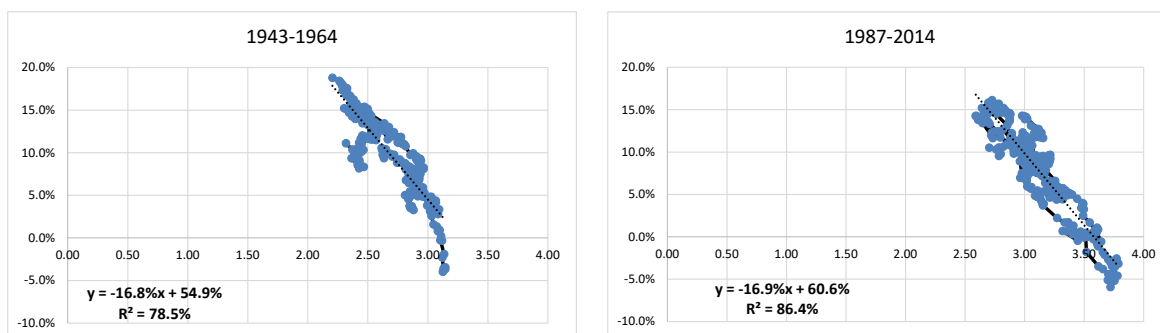
The apparent decline in the model's explanatory power is likely misleading. Indeed, it would be questionable to insist on preserving a single model with limited explanatory capacity, when the relationship between the Shiller P/E and market performance clearly serves as a kind of leading indicator of the prevailing market regime. There is, in fact, little reason to assume that this regime remains stable — something that the experience of distinctly different market phases tends to disprove (see below on the concept of “regime”).

As this article does not aim to provide an exhaustive analysis of the CAPE, we will limit ourselves here to presenting the four most recent market regimes that we have identified, seemingly prevailing between 1939 and 2014, based on this analytical tool — the Shiller P/E.

In each of the charts below (see Figure 5), the vertical axis shows the annualized 10-year ex-post performance of the S&P Composite-500 index, adjusted for inflation (based on the Consumer Price Index) and assuming theoretical reinvestment of dividends. The horizontal axis shows the logarithm of the CAPE prevailing at the time of investment. For clarity, the CAPE observation is recorded at the beginning of the month, while the investment is assumed to occur at the end of the same month (source: Robert Shiller's website).

Figure 5: Identifiable Market Phases through the Shiller CAPE:





As can be seen, from one regime to the next, the slope of the regressions remains relatively stable, corresponding to a marginal effect of approximately -16% for changes in the logarithm of the CAPE ratio. In contrast, the intercept ranges from 43% over the 1965–1986 period to more than 60% in the most recent phase (1987–2014). Finally, each market regime appears to fall within a distinct range of average returns and CAPE logarithms. It is also worth noting that these logarithmic values have tended to shift to the right during the latest phase, reflecting an upward drift in valuation multiples.

What the model does not explain is why regime shifts occur—these are only identified ex post, once they have already materialized. Moreover, the duration of these market phases is not constant. The most recent phase appears to have lasted at least thirty years (which coincides with the relative stationarity of the ex-ante IRR observed since the early 1990s), while the two preceding ones each lasted about twenty years. To avoid drawing hasty conclusions, it should be noted that the first market phase shown here lasted only three years, roughly from the outbreak of World War II to the liberation of Stalingrad—i.e., the period during which the outcome of the conflict was most uncertain from the Allied perspective. Based on our analysis, it seems that prior to World War II, market phases were shorter—closer to a decadal rhythm—which aligns with Benjamin Graham’s (1934) empirical observations and recommendations regarding the appropriate time horizon for assessing corporate profitability.

The notions of regime and market phase invoked here are rooted in empirical observation and intuition. In hindsight, one can easily point to specific structural or cyclical events to account for transitions between phases. With the exception of the 1939–1943 episode, we refrain from cataloging historical events, although most readers will readily associate the years 1965–1966 and 1987 with major market shifts. For our purposes, observations were grouped with the aim of maximizing the coefficient of determination within each cluster.

Within each market phase—assuming a stable valuation convention—equity returns appear to be predictable. Transition phases, however, cannot be anticipated solely on the basis of the CAPE ratio. This is precisely the added value of the dynamic model presented in the main body of the

article: it offers the beginning of a framework for explaining the entire period under study, including the regime shifts themselves.

Appendix 2: A Justification of the Ex-Ante IRR Under Constant Uncertainty and Volatility

In a market regime characterized by stable radical uncertainty and return volatility, it appears reasonable to expect a certain level of return from a risky asset such as an equity, based on a Brownian diffusion model. Indeed, if one assumes that stock market returns follow a normal distribution with mean μ and standard deviation σ , then it can be shown that the terminal value of the index, V_t , follows a log-normal distribution such that:

$$\ln(V_T) \sim N(\mu + \ln(V_0), \sigma) \quad 9)$$

We thus derive the expression for the logarithmic return²²:

$$\ln\left(\frac{E(V_T)}{V_0}\right) = \mu + \frac{\sigma^2}{2} \quad 10)$$

- The left-hand side represents the return of the index in continuous time. It corresponds to the logarithmic return of the investment, which we will denote as $E(R_L)$. If the index is equal to 1 at $t = 0$, and the expected annually compounded return in discrete time is R % at $t = T$, then;

$$E(R_L) = \ln(1 + R) = \ln\left(\frac{E(V_t)}{V_0}\right) \Leftrightarrow E(V_t) = V_0 \times (1 + R) \quad 11)$$

- The right-hand side of Equation (10) expresses the decomposition of expected return into two components: a deterministic component, μ , and a stochastic component, $\sigma^2/2$. The central tendency parameter μ is also referred to as the drift.

If we take, for example, a target of 6.4% for the expected logarithmic return of the U.S. stock market²³ — that is, the relatively stable level observed over the past decade according to the Fairness Finance model — along with a volatility of index returns of 13.4% over the period, and

²² See Aftalion (2008), Ch. 1.

²³ The expected return of 6.4% (6.6% in discrete time) is lower than the gross IRR predicted by the model, which lies between 8% and 9% (in discrete time). The gap between the two rates corresponds to the effect of the optimism bias that affects analysts' cash flow forecasts, according to the Fairness Finance model.

a dividend yield close to 1.8%,²⁴, then the central trend parameter μ would be approximately 3.7%: $0,037 = 0,064 - 0,134^2/2 - \ln(1,018)$.

Since 2014, it has been observed that the portion of the index's growth not attributable to its volatility—approximately 3.7%—closely matches the growth rate of the equity capital of the companies making up the index, which reflects the expansion of productive capacity and effective demand²⁵. From a macroeconomic perspective, and taking into account the volatility of stock prices, there may thus be a certain logic to the market having delivered a logarithmic return of at least 6.4%. In fact, the actual return reached 8.6%. This outperformance relative to the expected return could partly reflect a slight decline in the IRR. With an average risk-free rate of 1.9%—corresponding to the yield on 10-year T-Bonds—the equity risk premium would range between 4.5% ex-ante and 6.7% ex-post, a level that remains high by the standards of the consumption-based CAPM.

While the risk-free rate reached historically low levels in the years following the 2008 crisis, it is possible that this period marked a sort of floor in the IRR, which would explain the apparent increase in the observed equity risk premium during that time.

This would constitute a one-off and unprecedented episode, as long-term perspective suggests that fundamental uncertainty takes precedence over both the pace of capital accumulation and stock market volatility in explaining the level of IRR (see Figure 3 in the main text).

Appendix 3: Proximity Between the Ex-Ante IRR and the CAPE

At the aggregate level of the market index, the proximity between the IRR and the CAPE can be highlighted through the Gordon–Shapiro formula applied to the forward-looking normative cash flow of all the firms in the index.

If E denotes the aggregate earnings generated on average over a rolling ten-year period by the firms in the index portfolio—as is done when computing the CAPE—then the cash flow (CF) that would justify the aggregate value (the capitalization of the index) could, as a first approximation, be viewed as that of a single firm:

²⁴ Dividends paid out reduce the growth of the index value by an equivalent amount. They account for approximately 1.8% of the value of the stocks included in the index: $E(TRI) = E(FP) + E(dy) + \sigma^2/2$, where FP denote the growth rate of book equity, dy the dividend yield (i.e., dividends divided by price), and σ^2 the variance of price returns.

²⁵ For further details, see Roland Clère (2025).

$$E(CF) = p \times (\alpha \cdot E - g \cdot B) \quad 12)$$

Where:

- α is a coefficient generally slightly greater than 1. Indeed, the normative earnings E must on average be adjusted upward to account for the growth in productive capacity and capital over the past ten years, as well as for the one-year growth of those earnings from the discounting date (at an annualized rate).
- g is the perpetual growth rate of productive capacity and equity (assuming constant leverage). The term $g \cdot B$ thus refers to the portion of earnings that must be retained to maintain constant leverage and, more broadly, to preserve the prudential ratios of the underlying firms and to cover the likely inflation of investment costs (under constant capital intensity). As a result, the payout ratio is assumed to be structurally below 1 and decreasing as growth increases.
- Since the average reported earnings E essentially reflect the performance of surviving firms—due to both the size-based composition of the index and the quarterly reporting window chosen by S&P to compute E —it is likely that $(\alpha \cdot E - g \cdot B)$ represents a conditional expectation of CF in the case of survival (i.e., a truncated expectation). The mathematical expectation of CF must therefore plausibly be obtained by applying a coefficient p slightly below 1, given that the default risk of the largest firms contributing most to the index calculation is likely reduced, and the portfolio can potentially be rebalanced well before any default by one of these large contributors occurs (with rare and notable exceptions).

Consequently, if we let P denote the capitalization of the index and k the IRR that equates it with the present value of future cash flows, then, based on the Gordon–Shapiro formula, we have:

$$P = \frac{p(\alpha \cdot E - g \cdot B)}{k - g} \quad 13)$$

From this, we deduce the functional relationship between the IRR and the CAPE:

$$k = p \cdot \alpha \left(\frac{P}{E}\right)^{-1} + g - p \cdot g \left(\frac{P}{B}\right)^{-1} \quad 14)$$

where $(P/E)^{-1}$ is the inverse of the CAPE and $(P/B)^{-1}$ the inverse of the price-to-book multiple.

If all the parameters of Equation (14) were fixed over a given horizon, the relationship between

k and the CAPE would be affine. However, this is not the case due to the variability of g, p, α , and especially of the price-to-book multiple, which depends on expected ROE.

An additional source of imperfect correlation between the two parameters arises from the way the market IRR is calculated. According to the CAPM, the required return on the market portfolio equals the weighted average of the individual returns of its components, weighted by their capitalizations. Consequently, k is not computed as in Equation (14), as if the market were a single aggregated firm. This parameter therefore reflects structural sectoral divergences—notably in leverage, payout ratios, and growth trajectories—that are not captured in Equation (14) and that contribute to a partial decoupling between the CAPE (whose earnings are computed in aggregate) and the market IRR, which is a capitalization-weighted average of individual IRRs.

Finally, it should be noted that even if Equation (14) were applied to each firm, it would not yield exactly the IRR as calculated in practice by the providers who publish it. Measurement gaps exist between the market's expectations and the published IRR, which relies on analyst forecasts that do not adjust in real time. This can generate temporary discrepancies at the aggregate level—through the weighting mechanism—or even transitory biases during market phase shifts (some permanent biases can, however, be anticipated and partially corrected).

Thus, based on quarterly observations over a 10.5-year period between late 2014 and June 2025, the conditional²⁶ market IRR estimated by the Fairness Finance model under survival, $E(k|S)$, showed negative correlations of 88% in levels and 77% in first differences with Robert Shiller's CAPE. By contrast, the unconditional IRR, $E(k)$, once adjusted by subtracting the default premium from the conditional rate, exhibited only a -14% correlation in levels and a +24% correlation in first differences with the CAPE. As can be seen, accounting for default risk produces a very strong decoupling between the required IRR (which is unconditional) and the CAPE, whose denominator essentially represents a lagged value of the conditional mathematical expectation of expected net earnings.

This examination of correlations can be extended by a Granger causality analysis between the conditional required return, $E(k|S)$, and the CAPE ratio, using quarterly data covering the period from December 2014 to June 2025. To ensure stationarity of the series, they were transformed into first differences and subjected to augmented Dickey–Fuller tests. The results are presented below: for each number of time lags retained (in quarters) between the dependent and explanatory

²⁶ By conditional IRR, $E(k|S)$, we mean the expected required return under the assumption of survival of the firms in the index; the unconditional IRR, $E(k)$, also incorporates the probability of default, such that: $E(k) = (1 - \pi) \times E(k|S) + \pi \times k_d$, where π denotes the probability of default and k_d the return in the event of default, which for shareholders is generally close to zero.

variable, the table reports the probability associated with the null hypothesis of no causal relationship.

Granger Causality Test Results (p-values)

Lags	$\Delta E(k S) \rightarrow \Delta CAPE$	$\Delta CAPE \rightarrow \Delta E(k S)$
1	0,43%	6,7%
2	2,20%	4,24%
3	0,09%	0,85%
4	0,03%	0,88%
5	0,15%	1,15%
6	1,47%	3,06%
7	15,2%	16,7%
8	7,7%	22,9%
9	14,9%	23,3%
12	79,6%	90,5%

These results confirm significant short-term bidirectional causality (up to six quarters) but highlight a more robust relationship in the direction $\Delta E(k|S) \rightarrow \Delta CAPE$. In other words, variations in the conditional required return appear as the leading variable, strongly influencing changes in the valuation ratio, while the reverse effect exists but remains more limited.

In conclusion, the following hypothesis can be formulated: the implied IRR and the CAPE seem to play complementary but distinct roles within the valuation convention. The former, as the ex-ante required return, would constitute the structuring variable that conditions valuation; the latter would follow from it but would not act as a symmetric cause. Rather, the CAPE would operate as a signal—or even as a psychological threshold—framing the collective perception of valuation levels and thereby contributing to the maintenance of the convention, without determining its center of gravity.

Appendix 4: Principles for Constructing the Crisis Dummy Variable

In the modeling of one-year ex-post stock market performance based on the P/E ratio and its derivatives (see Chapter IV, §2), the momentum effect of the P/E ratio weakens sharply during the quarters coinciding with the peak phase of the 2008–2009 financial crisis. The same holds true for the mean-reversion effect captured by the variable $g\{P - 1\}$, which is a continuous indicator reflecting recent stock market performance prior to investment.

To capture the influence of the crisis on these two variables, a dummy variable, X_{is} , is introduced. It reflects the perception of systemic risk and the psychological threshold associated with the crisis, with an expected persistence beyond the initial shock.

For this purpose, the variable X_{it} is constructed from three parameters that we consider as potential indicators of a financial crisis. Published by the Federal Reserve Bank of St. Louis (FRB St. Louis) and taken from its FRED database, these are: (i) the VIX index of stock market volatility, calculated by Cboe from one-month index option prices; (ii) the Baa–Treasury credit spread, defined as the difference between the average yield on Baa-rated corporate bonds (Moody’s) and the yield on constant-maturity 10-year U.S. Treasury bonds; and (iii) the TED spread, measured as the difference between the 3-month LIBOR and the 3-month T-Bill yield, an indicator of risk and liquidity premia in the interbank market²⁷.

In a first step, these three variables are homogenized using a common procedure of centering and standardization over a rolling window. Thus, for each variable, at investment date t , we calculate (i) its rolling monthly mean, then (ii) the deviation of this mean from its five-year moving average, and finally (iii) standardize this deviation by the rolling standard deviation of the variable over the same five-year window. This standardization provides rolling-window z-scores expressing the relative position of each indicator with respect to its recent history, with all necessary information available at calculation date t . To smooth fluctuations, we then retain the rolling one-month mean, μZ , of these standardized scores.

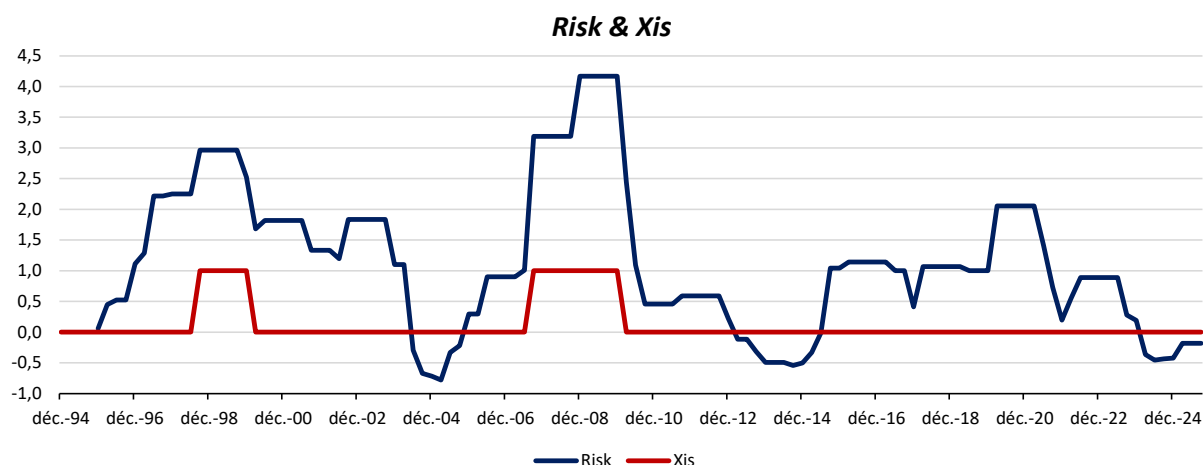
In a second step, from these three smoothed scores μZ , we construct a synthetic measure: the variable $m|\mu Z > Min$, defined as the average of the two highest smoothed scores at date t . This construction thus discards the lowest indicator and retains a measure of stress based on the two dimensions assumed to be most influential. At this stage, $m|\mu Z > Min$ is therefore a continuous variable.

From $m|\mu Z > Min$, we then derive the stepwise variable Risk. It corresponds to the rolling maximum of $m|\mu Z > Min$ over five consecutive quarters. It is thus assumed to reflect the perception of financial stress by incorporating a memory effect.

Finally, the dummy variable X_{it} is defined as equal to 1 when Risk exceeds the threshold of 2.5, which allows identification of the most intense systemic crisis episodes, representing about 10% of observations. This calibration of the variable—by maximizing the explanatory power of the model (R^2 , AIC)—is not intended to identify a universal threshold, but rather to illustrate a memory-with-threshold heuristic. Indeed, the choice of 2.5 optimizes the model fit and leads ex post to limited activation ($\approx 10\%$ of observations), consistent with the extreme nature expected of such a threshold. The principle therefore matters more here than the exact parameter values,

²⁷ Since March 2016, the TED spread series has no longer been published by the Federal Reserve Bank of St. Louis. We extend it using the spread between 90-day AA financial commercial paper and 3-month T-bills, which is representative of top-tier financial issuers.

which are necessarily sample-contingent and whose stability over time is not guaranteed²⁸. The variable is thus activated for ten consecutive quarters, from September 30, 2007 to December 31, 2009. It would also have been activated for six quarters, from September 1998 to December 1999, had this period been included in the scope of analysis.



In summary, the variable *Xis* provides a synthetic measure of systemic financial crisis, constructed from standardized market indicators and incorporating a memory effect. This persistence reflects a hysteresis mechanism, whereby the impact of a shock does not dissipate immediately but leaves a lasting imprint on the Risk variable. In practice, the dummy *Xis* formalizes a regime shift, distinguishing a normal state from a crisis state defined by an exogenous threshold. It is thus activated only during episodes of major financial stress—corresponding in the main analysis to the subprime crisis (2007–2009) and, counterfactually, to the Asian and Russian crises as well as the LTCM collapse (1998–1999). It should be noted that neither the Internet bubble (which was highly sectoral) nor the COVID-19 pandemic triggered activation of *Xis*, which seems to reflect the fact that neither, for different reasons, generated a potentially systemic financial crisis.

²⁸ It should be noted that a Markov switching model would produce a crisis probability, but a cutoff threshold would still need to be set—in practice chosen to maximize fit—which would amount to the same logic as the calibration adopted here with 2.5.